12th International ISAAC Congress

Volume of Abstracts

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Abstracts

Edited by P. Cerejeiras and U. Kähler. Prepared and typeset using LaTeX.

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Welcoming address

The ISAAC board, the Local Organising Committee and the Department of Mathematics at Imperial College London, are pleased to welcome you to the 12th International ISAAC Congress in Aveiro. The 12th International ISAAC congress continues the successful series of meetings previously held in the Delaware, USA (1997), Fukuoka, Japan (1999), Berlin, Germany (2001), Toronto, Canada (2003), Catania, Italy (2005), Ankara, Turkey (2007), London, UK (2009), Moscow, Russia (2011), Krakow, Poland (2013), Macao, China (2015), and Växjö, Sweden (2017).

The success of such a series of congresses would not be possible without the valuable contributions of all the participants.

We acknowledge the financial support for this congress given by

the *FCT–Fundação para a Ciência e a Tecnologia* (FCT), through the *Fundo de Apoio à Comunidade Científica* (FACC),

the *Universidade de Aveiro* (UA),

the *Center for Research and Development in Mathematics and Applications* (CIDMA),

and the *Departamento de Matemática da Universidade de Aveiro*.

Also, we wish to thank the many Portuguese companies who's support was crucial for the success of this event:

Nestlé Portugal, *Porto de Aveiro*, *Real Companhia Velha*, *Sociedade da Água Luso*, *TAP Air Portugal*, *The Navigator Company*, and *Turismo Centro de Portugal*,

Abstracts

Plenary talks

Statistical Estimation with Algebraic Structure: Statistical and Computational Considerations

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The massive datasets now commonplace in many fields of industry and science hold the promise of significantly changing our understanding of the world around us. Key to the fulfillment of this promise however, is the ability to perform computationally efficient statistical inference on such large datasets. This motivates one of the key questions in theoretical data science, to understand which statistical inference tasks are impossible, which are possible but computationally infeasible for large datasets, and which enjoy efficient estimation procedures.
In this talk, we will focus on an important class of estimation problems that are rich in algebraic structure. Motivated by problems in signal/image processing, and computer vision we will address the task of estimating a signal, image, tri-dimensional structure/scene from measurements corrupted not only by noise but also by latent transformations. Many such transformations can be described as a group acting on the object to be recovered. Examples include the Simulatenous Localization and Mapping (SLaM) problem in Robotics and Computer Vision, where pictures of a scene are obtained from different positions and orientations; Cryo-Electron Microscopy (Cryo-EM) imaging where projections of a molecule density are taken from unknown rotations, and several others.

Afonso Bandeira received his Ph.D. in Mathematics from Princeton University in 2015. Currently, he is a Professor of Mathematics and Data Science at the Courant Institute. His contributions to Mathematics cover a broad range with fundamental contributions to probability, harmonic analysis, numerical analysis, mathematical signal processing, graph theory, data science, and neural networks. Among other awards he received a Sloan Research Fellowship in 2018 and was selected as a 2015 DARPA Riser.

Analysis on fractal spaces and heat kernels

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We discuss elements of Analysis on singular metric spaces, in particular, on fractals, based on the notion of the heat kernel. Such spaces are characterized by two parameters: the Hausdorff dimension and the walk dimension, where the latter determines the space/time scaling for a diffusion process. We present various approaches to the notion of the walk dimension, including those via Besov function spaces and via Markov jump processes. We also discuss heat kernel bounds for diffusion and jump processes.

Alexander Grigor'yan received his PhD from Moscow State University in 1982 and his Doctor of Physycomathematical sciences by Moscow State University in 1989. He was a visiting scholar at Harvard University in 1993-1994 and moved to the Imperial College London until 2005. Since then he is Professor of Mathematics at University of Bielefeld in Germany. He made fundamental contributions in Geometric Analysis on Riemannian manifolds, graphs, and metric spaces. This in includes analysis of partial differential equations of elliptic and parabolic types on Riemannian manifolds, random walks on graphs, function theory and diffusion processes on fractal spaces. Major results are on heat kernel estimates, estimates on eigenvalues of Laplace and Schrödinger operators and long time behavior of random walks and diffusions. Furthermore, this also resulted in fundamental contributions to the theory of stochastic processes on manifolds, graphs,

and fractals. For his contributions he received the Whitehead prize in 1997 and the prize of the Moscow Mathematical Society in 1988.

Totally positive functions in sampling theory and time-frequency analysis

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Totally positive functions play an important role in approximation theory and statistics. In this talk I will present recent new applications of totally positive functions (TPFs) in sampling theory and time-frequency analysis.
(i) We study the sampling problem for shift-invariant spaces generated by a TPF. These spaces arise the span of the integer shifts of a TPF and are often used as a substitute for bandlimited functions. We give a complete characterization of sampling sets for a shift-invariant space with a TPF generator of Gaussian type in the style of Beurling.
(ii) A related problem is the question of Gabor frames, i.e., the spanning properties of time-frequency shifts of a given function. It is conjectured that the lattice shifts of a TPF generate a frame, if and only if the density of the lattice exceeds 1. At this time this conjecture has been proved for two important subclasses of TPFs. For rational lattices it is true for arbitrary TPFs. So far, TPFs seem to be the only window functions for which the fine structure of the associated Gabor frames is tractable.
(iii) Yet another question in time-frequency analysis is the existence of zeros of the Wigner distribution (or the radar ambiguity function). So far all examples of zero-free ambiguity functions are related to TPFs, e.g., the ambiguity function of the Gaussian is zero free.

Karlheinz Gröchenig is an Austrian Mathematician at the Faculty of Mathematics at University of Vienna. He made his Ph.D. degree sub auspiciis praesidentis at the University of Vienna in 1985. After positions at the McMaster University and the University of Connecticut he returned to Vienna in 2006. He made fundamental contributions in Harmonic analysis, time-frequency analysis, wave- let theory, (non-uniform) sampling theory, pseudodifferential operators, Banach algebras and their symmetry, non-commutative harmonic analysis. Among many other awards he received the Marie-Curie Excellence Award of the European Union in 2004.

Planar orthogonal polynomials and related point processes: random norm matrices and arithmetic jellium

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In the random normal matrix model, the correlation kernel is expressed as a sum over orthogonal polynomials. The spectra of such normal matrices typically fill a region, the "spectral droplet". The local behavior of the eigenvalues near the edge of the droplet has attracted attention, and it was conjectured that the behavior is asymptotically governed by the error function kernel, for smooth boundary points. Here, we resolve this conjecture, assuming the whole boundary is smooth. The approach goes through developing a new asymptotic expansion for the orthogonal polynomials, which goes beyond the classical work of Szegö, Carleman, and Suetin. This allows us to also consider ensembles where the correlation kernel has gaps in the sense of summing only over some of the orthogonal polynomials. If we sum over an arithmetic progression we obtain "arithmetic jellium", a compound with orbifold structure generating a representation of $SU(1,1)$. This reports on joint work with Aron Wennman.

Håkan Hedenmalm received his Ph.D. in Mathematics Uppsala University in 1985. Prior to joining the Royal Institute of Technology, he served in the Department of Mathematics at Lund University until 2002. He made major contributions in a variety of fields including the theory of Bergman spaces and associated reproducing kernels, Several complex variables, the theory of biharmonic operators, the theory of Dirichlet

series, quasiconformal mappings, random matrices and polyanalytic determinantal processes. For his contributions he received the Wallenberg prize in 1882, the Göran Gustafsson Prize in 2002 and the Eva and Lars Gårding Prize in 2015. In 1997 he was elected to the Royal Physiographic Society in Lund and in 2018 to the Royal Norwegian Society of Sciences and Letters.

Recent progress in the theory of minimal surfaces

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A long standing problem in geometry, conjectured by S.-T. Yau in 1982, is that any any 3-manifold admits an infinite number of distinct minimal surfaces. The analogous problem for geodesics on surfaces led to the discovery of deep interactions between dynamics, topology, and analysis. The last couple of years brought dramatic developments to Yau's conjecture, which has now been settled due to the work of Marques-Neves and Song.
I will survey the history of the problem, its significance, and all the contributions made throughout the last 30 years.

André Neves received his Ph.D. in Mathematics from Stanford University in 2005. He has a Licenciatura degree from the Instituto Superior Técnico (1999). Prior to joining the faculty of the University of Chicago in 2016 he was a Professor at the Imperial College. In 2012, together with Fernando Marques, he proved that the least bended torus in space is the Clifford torus and, in doing so, solved the Willmore conjecture. For his contributions he received numerous awards, including the Philip Leverhulme Prize in 2012, the LMS Whitehead Prize in 2013, the Royal Society Wolfson Research Merit Award in 2015, a New Horizons in Mathematics Prize, Oswald Veblen Prize in 2016, and a Simons Investigator Award in 2018. His current research interests include Riemannian geometry, scalar curvature, Min-Max Theory, Mean Curvature Flow, and General Relativity.

Multidimensional inverse scattering problem

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We give a short review of old and recent results on the multidimensional inverse scattering problem for the Schrödinger equation. A special attention is paid to efficient reconstructions of the potential from scattering data which can be measured in practice. In this connection our considerations include reconstructions from non-overdetermined monochromatic scattering data and formulas for phase recovering from phaseless scattering data. Potential applications include phaseless inverse X-ray scattering, acoustic tomography and tomographies using elementary particles. This talk is based, in particular, on results going back to M. Born (1926), L. Faddeev (1956, 1974), S. Manakov (1981), R.Beals, R. Coifman (1985), G. Henkin, R. Novikov (1987), and on more recent results of R. Novikov (1998 - 2019).

Roman Novikov received his Ph.D. in Mathematics from M.V. Lomonosov Moscow State University in 1990 under the supervision of S.P. Novikov and his Doctor of Physyco-mathematical sciences by the St. Petersburg Branch of V.A. Steklov Mathematical Institute in 1998. Currently, he is Directeur de recherche au CNRS at CMAP - Centre de Mathématiques Appliquées, École Polytechnique, France. He made fundamental contributions in the theory of inverse problems, especially in the theory of inverse scattering and tomography, discrete function theory, and the theory of Riemann-Hilbert problems.

Self-similar solutions to the derivative nonlinear Schrödinger equation

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A class of self-similar solutions to the derivative nonlinear Schrödinger equations is studied. Especially, the asymptotics of profile functions are shown to posses a logarithmic phase correction. This logarithmic phase correction is obtained from the nonlinear interaction of profile functions. This is a remarkable difference from the pseudo-conformally invariant case, where the logarithmic correction comes from the linear part of the equations of the profile functions. This talk is based on my recent jointwork with Kazumasa Fujiwara (Tohoku) and Vladimir Georgiev (Pisa).

Tohru Ozawa has been a Professor of Mathematics at Waseda University in Tokyo, Tokyo, Japan, since 2008. He graduated with honors from the Department of Physics at Waseda University in 1984. He received an M.S. degree in 1986 and Ph.D. degree in 1990 from RIMS, Kyoto University. He held a Full Professor position from 1995 to 2008 at Hokkaido University in Sapporo. In 1998, he was awarded a Spring Prize from the Mathematical Society of Japan for his pioneering work on the nonlinear Schrödinger equation. His research interests include nonlinear Schrödinger equations (scattering theory, Strichartz estimates, analyticity and smoothing properties of solutions, derivative coupling, etc.), nonlinear hyperbolic equations (Klein-Gordon equation, Dirac equation, systems of wave equations, Strichartz estimates, nonrelativistic limit, self-similar solutions, etc.), and function spaces (embedding theorems, interpolation inequalities, characterizations of fractional Sobolev spaces over Lie groups, etc.).

Inverse Problems and the Nonlinear Fourier Transform

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Electrical impedance tomography (EIT) is an emerging medical imaging method. It is based on probing the human body with harmless electric currents fed through electrodes on the skin. The voltages appearing on the electrodes are measured, and the aim of EIT is to recover the internal distribution of electric conductivity. The resulting image can be used for diagnosing stroke or assessing the lung function of cystic fibrosis patients. The mathematical model of EIT is the inverse conductivity problem introduced by Alberto Calderón in 1980. It is a generic example of an ill-posed inverse boundary value problem, where one tries to reconstruct a PDE coefficient from a Dirichlet-to-Neumann map. This reconstruction task is highly sensitive to modelling errors and measurement noise, and therefore requires regularised solution.

A mathematically satisfying regularisation approach is offered by a nonlinear Fourier transform, based on Complex Geometric Optics solutions introduced by John Sylvester and Gunther Uhlmann in 1987. A low-pass filter applied on the nonlinear frequency domain enables robust real-time EIT imaging, with cutoff frequency determined by the amplitude of measurement noise. This imaging method is based on solving a D-bar equation and is connected to the theory of pseudoanalytic functions.

There are further interesting possibilities arising from the use of the nonlinear Fourier transform. An added one-dimensional Fourier transform leads to singularity propagation along two-dimensional leaves, according to the Duistermaat-Hörmander theory of complex principal type operators. This can be used in EIT imaging for recovering boundaries between tissues and organs.

Furthermore, the nonlinear Fourier transform can be used for linearising the Novikov-Veselov equation, a (2+1) dimensional generalisation of the KdV equation.

Based on these examples it is safe to say that the nonlinear Fourier transform is a versatile tool applicable to very different problems. It surely holds more secrets yet to be revealed.

Samuli Siltanen received his Ph.D. in Applied Mathematics from Helsinki University of Technology in 1999. Currently, he is Professor of Industrial Mathematics at University of Helsinki and Team leader in the Finnish Centre of Excellence in Inverse Modelling and Imaging. He made major contributions in the area of inverse problems, especially in the conductivity problem and inverse scattering. He also focuses on practical and

industrial implementations and is holder of several patents. Among other prizes he received the J.V. Snellman Prize in 2018.

Hardy inequalities on homogeneous groups

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In this talk, we discuss Hardy inequalities and closely related topics from the point of view of Folland and Stein's homogeneous (Lie) groups. The place where Hardy inequalities and homogeneous groups meet is a beautiful area of mathematics with links to many other subjects. In short, our main idea is consistently working with relations of quasi-radial derivatives and the Euler operators, so from these relations follow various Hardy type inequalities with sharp constants on homogeneous groups. While describing the general theory of Hardy type inequalities in the setting of general homogeneous groups, we pay particular attention to the special class of stratified groups. In this environment, the theory of Hardy inequalities becomes intricately intertwined with the properties of sub-Laplacians and subelliptic partial differential equations. To demonstrate applications of the theory we present solutions of two previously known conjectures. Particularly, we discuss the Badiale-Tarantello conjecture and the conjecture on the geometric Hardy inequality in a half-space on the Heisenberg group with a sharp constant. The present talk is partially based on our recent open access book (with the same title) with Michael Ruzhansky.

Durvudkhan Suragan received his Ph.D. in Mathematics from Al-Farabi Kazakh National University in 2013. Prior to joining Nazarbayev University in April 2018, he served in the Department of Mathematics at Imperial College London as a research associate and at Institute of Mathematics and Mathematical Modeling (Almaty) as a leading reseacher. His current research interests include analysis on Lie groups, applied mathematics (mathematical physics), PDE, potential theory and spectral geometry. In 2013, Suragan was awarded the Konayev prize for young scientists (in Kazakhstan) for the best work in the field of natural sciences. In 2018, Suragan won the Ferran Sunyer i Balaguer Prize for his (with Professor Michael Ruzhansky, Imperial College London) monograph: Hardy inequalities on homogeneous groups, Progress in Mathematics, Vol. 327, Birkhauser, 2019. xvi+588pp.

Sessions

S.1 Applications of dynamical systems theory in biology

Organisers
Torsten Lindström

Scope of the session: In this session we primarily accept talks that are using dynamical systems theory in order to analyze various models that arise in biological applications. The models analyzed may be mechanistically formulated, fitted to data, deterministic, or stochastic. Various relations between such models that arise in different modeling approaches and under different simplifying assumptions can be analyzed. Possible biological applications can include ecology, epidemiology, pharmacokinetics, evolution, physiology, pattern formation, and resource distribution, but are not limited to these topics.

—Abstracts—

Backward bifurcation in SEIRS model with satuation incidence rate and treatment

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Abstract

Nonlinear analogue of the May-Wigner instability transition

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How many equilibria will a large complex system, modelled by $N \gg 1$ randomly coupled autonomous nonlinear differential equations typically have? How many of those equilibria are stable being local attractors of nearby trajectories? Such questions arise in many applications, notably in stability analysis of ecological communities as originally posed by Robert May in 1972. Recently they has been partly answered within the framework of a model introduced and analysed in Y.V. Fyodorov, B.A. Khoruzhenko (2016), and G. Ben Arous, Y.V. Fyodorov, and B.A. Khoruzhenko (under preparation) exploiting recent insights in the theory of large random asymmetric matrices. We show that with increased interaction strength such systems generically undergo an abrupt transition from a trivial phase portrait with a single stable equilibrium into a topologically non-trivial regime of 'absolute instability' where equilibria are on average exponentially abundant, but typically all of them are unstable, unless the dynamics is purely gradient. When interactions increase even further the stable equilibria eventually become on average exponentially abundant unless the dynamics is purely solenoidal. We further calculate the mean proportion of equilibria which have a fixed fraction of unstable directions.

Global bifurcations of limit cycles and strange attractors in multi-parameter polynomial dynamical systems

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We develop new bifurcational geometric methods based on the Wintner–Perko termination principle for the global bifurcation analysis of planar multi-parameter polynomial dynamical systems. Using these methods, we present, e. g., a solution of Hilbert's Sixteenth Problem on the maximum number and distribution of limit cycles for the Kukles cubic-linear system, the general Liénard polynomial system with an arbitrary number of singular points, a Leslie–Gower system which models the population dynamics in a real ecological or biomedical system, and for a reduced Topp system which models the dynamics of diabetes. Applying a similar approach, we study also three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenarios in Lorenz type systems connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of limit cycles.

Age- and size-structured models of population dynamics: optimal control and sustainability

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Optimal control of population dynamics in forestry, fishery, agriculture and environmental sciences will be discussed. The focus will be on arising differential and integral dynamic population models with two-dimensional distributed controls. Such models describe a population as a deterministic dynamic system with heterogeneous elements, whose performance depends on the current time and an additional independent variable (age or size).
Two models, the age-structured Lotka-McKendrik model of population dynamics and a nonlinear size-structured model of rational forest management, will be considered. In forestry and fishery, the link between the size and age of population individuals is rather weak and corresponding models use the individual size rather than its age as a variable, which leads to size-structured (physiologically structured) models. The models have been employed to solve various practical problems that include, but are not limited to, finding optimal harvesting strategies and exploring sustainable development under changing environmental conditions. The optimal control, steady-state analysis, and bifurcation analysis will be presented. Applied interpretation of the obtained outcomes will be provided.

Two-dimensional differential delay systems as mathematical models of physiological processes

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We study dynamical properties of solutions of two-dimensional differential systems with delay which serve as mathematical models of several physiological processes in human body. One such model describes the intracellular circadian rhythm generator; it was introduced in 1999 by Scheper et al. in the Journal of Neuroscience. Another one describes oscillations in a glucose-insulin interaction model with time delay; its several versions were introduced by different researchers who studied various aspects of this physiological phenomenon. Mathematically the models can be incorporated by a system of delay differential equations of the form

$$(1)\quad \begin{aligned} x'(t) &= -\alpha x(t) + f(x(t), y(t), x(t-\tau), y(t-\sigma)) \\ y'(t) &= -\beta y(t) + g(x(t), y(t), x(t-\tau), y(t-\sigma)), \end{aligned}$$

where nonlinearities f and g are continuous real-valued functions, decay rates α, β are positive, and delays τ, σ are non-negative with $\tau + \sigma > 0$. Sufficient conditions for the existence of periodic solutions in system (1) are established. The nonlinearities f and g are further assumed to satisfy either positive or negative feedback condition with the overall negative feedback in the system. The stability of the unique equilibrium and oscillation of all solutions about it are studied and derived in terms of the characteristic equation of the linearized system. The instability of the equilibrium together with a one-sided boundedness of either f or g lead to the existence of periodic solutions. The analysis of system (1) uses some of the recent results established for higher order differential delay equations.

On the limit configuration of four species strongly competing systems

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According to the competitive exclusion principle (also known as Gause's law), many competing species cannot coexist under very strong competition, but when spatial movements are permitted more than one species can coexist thanks to the segregation of their habitats. From a mathematical viewpoint the determination of the configuration of the habitat segregation is an interesting problem which can be modelled by an optimal (in a suitable sense) partition of a domain. In recent papers the problem is studied modelling the interspecies competition with a large interaction term in an elliptic system of partial differential equations inspired by classical models in populations dynamics.
In this talk we analysed some qualitative properties of the limit configuration of the solutions of a reaction-diffusion system of four competing species as the competition rate tends to infinity. Large interaction induces the spatial segregation of the species and only two limit configurations are possible: either there is a point where four species concur, a 4-point, or there are two points where only three species concur. We characterized, for a given datum, the possible 4-point configuration by means of the solution of a Dirichlet problem for the Laplace equation.
This is a joint work with Eugenio Montefusco (Sapienza University, Rome, Italy).

Applications of optimal control theory to cholera

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Applications of optimal control theory to cholera Abstract: We propose and study several mathematical models for the transmission dynamics of some strains of the bacterium *Vibrio cholerae*, responsible for the cholera disease in humans. Control functions are added with the purpose to obtain different optimal control problems. Their study allow us to determine the best way to curtail the spread of the disease. Such models are applied to the cholera's outbreak of Haiti (2010-2011) and Yemen (2017-2018).

Destabilization, stabilization, and multiple attractors in saturated mixotrophic environments

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In this talk we elucidate the dynamical consequences of the invasion of mixotrophs by the use of a model that is a limiting case the chemostat possessing explicit resource dynamics modeling. The model is a hybrid of a competition model describing the competition for resources between an autotroph and a mixotroph and a predator-prey model describing the interaction between the autotroph and a herbivore.
Mixotrophic interactions are a strong component in harmful algae blooms, Hallenrath-Lehmann et. al. (2015) and the purpose of this paper is not predicting recurrent harmful algae blooms. Instead we aim at understanding the environmental conditions allowing for mixotrophic invasions and their dynamical consequences.
It is possible for a mixotroph to invade both autotrophic environments and environments described by interactions between autotrophs and herbivores. The interaction between autotrophs and herbivores might be in equilibrium or cyclic. Our first conclusion is that it is possible for an invading mixotroph to both stabilize and destabilize autotrophs-herbivore dynamics, depending on the environmental conditions and the properties of the invading mixotroph.
Our second conclusion is that environmental conditions allowing for multiple attractors after mixotrophic invasion exist. Such initial value dependent behavior may be the consequence both after an invasion in a completely autotrophic environment and in both cyclic and equilibrium autotroph-herbivore environments.

The inverse problem on tree graph with attached masses from the point of view of neurobiology

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The model is based on the practical problem of dendritic trees formed by neurons of the central nervous system. The activity of neurons is accompanied by significant variations in electrical and physical parameters. Information is generated through an electrical impulse. The state of the system is described on a quantum graph [Avdonin S., Kurasov P., 2008] with differential operators on edges and the Kirchhoff-Neumann agreement conditions at the vertices. Unlike the staining and visualization methods for evaluating many dendritic properties, our approach is analytically rigorous and relies on the study of changes in currents. In contrast to [Stephenson E., Kojouharov H., 2018], we have given a mathematical justification based on an inverse problem for a parabolic equation on a tree graph. We expand the theory of neural cables by the BC method; we prove sufficient conditions for the identification of a priori parameters; new theorems on regularity and controllability in the inverse problem for a heat equation with memory on a tree graph with attached masses in internal vertices. On any given branch of the dendrite (edges e_i of the graph tree) there are sources of N_i connecting with other cells, where $N_i = 0, 1, 2, \ldots$ and N_i is a finite number. The obtained results on the recovering of sources allow us to study the functional activity of neurons in the process of influence of skin receptors.

Identification algorithm for differential equation with memory in biomathematical problems

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The paper is devoted to mathematical models of sensory physiology which lead to different class of inverse problems. Pacinian corpuscles are skin receptors that play a major role in sense of touch (Bell, 1994; Bell, 1999). A model for the membrane potential for the encapsulated nerve ending can be described by parabolic equation with memory on tree graph with attached masses. The coefficient m(x) of this equation is linearly related to the mechanical strain felt by the corpuscles nerve due to a local stimulus applied on the skin surface above the receptor. This problem boils down to constructing an algorithm for identifying the coefficient of a parabolic equation with memory on a tree graph with masses attached to internal vertices. We recover the m(x) coefficient by modifying the boundary control method [Avdonin, Kurasov 2008], and develop an algorithmic approach to estimate m(x) numerically. We develop methodology of [Avdonin, Bell, 2015] to identify edge radii, lengths, and conductances for our class of graphs. It is allows us to develop an algorithmic implementation to estimate these quantities. Many results concerning inverse problems for PDEs on graphs dealt mostly with the hyperbolic type equations. Our identification research involves applications to parabolic equations, so our response operator will be the Neumann-to-Dirichlet rather than Dirichlet-to-Neumann. What is more important, the matching Kirchhoff condition takes a slightly different form in some of these cases.

From Lotka-Volterra systems to Polymatrix Replicators

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Consider a population divided in a finite number of groups, each one with a finite number of strategies, where interactions between individuals of any two groups are allowed, including the same group. This model is designated as the polymatrix game. The system of differential equations associated to a polymatrix game, introduced recently by Alishah and Duarte in 2015 and designated as polymatrix replicator, form a simple class of ordinary differential equations defined on prisms given by a product of simplexes. This class of replicator dynamics contains well known classes of evolutionary game dynamics, such as the symmetric and asymmetric replicator equations, and some replicator equations for n-person games. As J. Hofbauer proved in 1981 the replicator equation is in some sense equivalent to the Lotka-Volterra (LV) system, independently introduced in 1920s by A. J. Lotka and V. Volterra. The LV system is perhaps the most widely known system used in scientific areas as diverse as biology, physics, chemistry, and economy. In this talk I will present the definition of polymatrix replicator, some basic properties, and some results about the dynamics and the inferences we can make about the associated polymatrix game.

Global dynamics of a new delay logistic equation arisen in cell biology

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The delayed logistic equation (also known as Hutchinson's equation or Wright's equation) was originally introduced to explain oscillatory phenomena in ecological dynamics. While it motivated the development of a large number of mathematical tools in the study of nonlinear delay differential equations, it also received criticism from modellers because of the lack of a mechanistic biological derivation and interpretation. Here we propose a new delayed logistic equation, which has clear biological underpinning coming from cell population modelling. This nonlinear differential equation includes terms with discrete and distributed delays. The global dynamics is completely described, and it is proven that all feasible nontrivial solutions converge to the positive equilibrium. The main tools of the proof rely on persistence theory, comparison principles and an L2-perturbation technique. Using local invariant manifolds, a unique heteroclinic orbit is constructed that connects the unstable zero and the stable positive equilibrium, and we show that these three complete orbits constitute the global attractor of the system. Despite global attractivity, the dynamics is not trivial as we can observe long-lasting

transient oscillatory patterns of various shapes. We also discuss the biological implications of these findings and their relations to other logistic type models of growth with delays.

Bifurcations on Invariant Tori in Predator-prey Models with Seasonal Prey Harvesting

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We study bifurcations in predator-prey systems with seasonal prey harvesting. First, when the seasonal harvesting reduces to constant yield, it is shown that various kinds of bifurcations, including saddle-node bifurcation, degenerate Hopf bifurcation, and Bogdanov-Takens bifurcation (i.e., cusp bifurcation of codimension 2), occur in the model as parameters vary. The existence of two limit cycles and a homoclinic loop is established. Bifurcation diagrams and phase portraits of the model are also given by numerical simulations, which reveal far richer dynamics compared to the case without harvesting. Second, when harvesting is seasonal (described by a periodic function), sufficient conditions for the existence of an asymptotically stable periodic solution and bifurcation of a stable periodic orbit into a stable invariant torus of the model are given. Numerical simulations, including bifurcation diagrams, phase portraits, and attractors of Poincaré maps, are carried out to demonstrate the existence of bifurcation of a stable periodic orbit into an invariant torus and bifurcation of a stable homoclinic loop into an invariant homoclinic torus, respectively, as the amplitude of seasonal harvesting increases. Our study indicates that to have persistence of the interacting species with seasonal harvesting in the form of asymptotically stable periodic solutions or stable quasi-periodic solutions, initial species densities should be located in the attraction basin of the hyperbolic stable equilibrium, stable limit cycle, or stable homoclinic loop, respectively, for the model with no harvesting or with constant-yield harvesting.

Stability and optimal control of a delayed HIV/AIDS-PrEP model

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Pre-exposure prophylaxis (PrEP) consists in the use of an antiretroviral medication to prevent the acquisition of HIV infection by uninfected individuals and has recently demonstrated to be highly efficacious for HIV prevention. In this work, we propose a delayed HIV/AIDS-PrEP model, given by a system of delayed differential equations. We analyze the case where the implementation of PrEP suffers a discrete time-delay and study the impact of the delay on the number of new HIV infections. The time delay describes mathematically barriers that block an effective implementation of PrEP, such as, stigma, cost and adherence. The model has two equilibrium points: disease free and endemic. A local and global stability analysis of the equilibrium points is performed, for any positive discrete time delay. In a final part, we formulate a delayed optimal control problem with the aim to determine the PrEP implementation strategy that minimizes the number of individuals with pre-AIDS HIV-infection, as well as the costs associated to PrEP. The theoretical results are illustrated through numerical simulations.

S.2 Complex Analysis and Partial Differential Equations

Organisers
Okay Celebi, Sergei Rogosin

Scope of the session: This session will be the combination of researchers whose interests are close to the Special Interest Group "Complex Analysis" and to the "Complex and functional analytic methods for differential equations".
This session supports the researches in classical areas of complex analysis such as initial and boundary value problems for (linear and nonlinear) complex differential equations and applications, boundary behavior of analytic and generalized analytic functions, conformal mappings, special functions in complex domains, entire and meromorphic functions, integral equations, as well as the modern directions in one- and several-dimensional complex analysis in which functional-analytic methods are also employed. Complex methods serve to construct explicit solutions to linear problems in the plane with generalizations to higher dimensions, while functional-analytic tools are applied to treat nonlinear equations with linear or nonlinear conditions. We also welcome the presentations which have applications in Mathematical Physics, Mechanics, Biology, Chemistry, Medicine, Economics etc. Applications in Fluid Dynamics, Composite Materials, Porous Media, Elasticity, Visco-Elasticity, Hydro-, Aero- and Thermo-Dynamics are the most considered.

—Abstracts—

Integral representations and some boundary value problems in Clifford analysis

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We study the Dirichlet and Neumann problems for Poisson equation in the unit ball using integral representation formulas in terms of Laplacian in the complex Clifford algebra $\mathbb{C}_m$. By introducing integral operators in Clifford analysis with their properties, we investigate the boundary value problems for higher-order Poisson equation.

Boundary Value Problems in Polydomains

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In this presentation we give a short survey of the boundary value problems in polydomains in the last decades. Firstly we develop an alternative method to derive integral representations for functions in $\mathbb{C}^n$. This unified method provides representations which are suitable to be employed in discussions for all linear boundary value problems. In the rest of the article we have improved some results obtained for Schwarz and Dirichlet type problems.
Subject classification: 32W10, 32W50; 31A10
Keywords; Polydisc, Schwarz problem, Riquier problem, complex partial differential equations

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Well-posedness and asymptotic results of 3D Burgers equation in Gevrey class

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We prove that the three-dimensional periodic Burgers equation has a unique global in time solution, in a critical Gevrey-Sobolev space. Comparatively to Navier-Stokes equations, the main difficulty is the lack of incompressibility condition. In our proof of existence, we overcome the bootstrapping argument, which was a technical step in a precedent proof, in Sololev spaces. This makes our proof shorter and gives sense of considering Gevrey class for a mathematical study to Burgers equation. To prove that the unique solution is global in time, we use the maximum principle. Energy methods, Sobolev product laws, compactness methods and Fourier analysis are the main tools.

Existence and Uniqueness Theorem for Cusped Kelvin-Voigt Elastic Plates in the Zero Approximation of the Hierarchical Models

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The talk is devoted to the homogeneous Dirichlet problem for the vibration problem of cusped viscoelastic Kelvin-Voigt prismatic shells in case of the zero approximation of the hierarchical models. The classical and weak setting of the problem are formulated. The spacial weighted functional spaces are introduced, which are crucial in our analysis. The coerciveness of the corresponding bilinear form is shown and uniqueness and existence results for the variational problem are proved. We describe in detail the structure of this spaces and establish their connection with weighted Sobolev spaces.

Derivation of system of the equations of equilibrium for shallow shells and plates with voids

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In the report the three-dimensional system of equations of equilibrium for solids with voids is considered. From this system of equations, using a reduction method of I. Vekua, we receive the equilibrium equations for the shallow shells. Further we consider the case of plates with constant thickness in more detail. Namely, the systems of equations corresponding to approximations $N = 0$ and $N = 1$ are written down in a complex form and we express the general solutions of these systems through analytic functions of complex variable and solutions of the Helmholtz equation. The received general representations give the opportunity to solve analytically boundary value problems.

The product-type operators from logarithmic Bloch spaces to Zygmund-type spaces

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The boundedness and compactness of a product-type operator, recently introduced by S. Stević, A. Sharma and R. Krishan,

$$T^n_{\psi_1,\psi_2,\varphi} f(z) = \psi_1(z) f^{(n)}(\varphi(z)) + \psi_2(z) f^{(n+1)}(\varphi(z)),\ f \in H(\mathbb{D}),$$

from the logarithmic Bloch spaces to Zygmund-type spaces are characterized, where $\psi_1, \psi_2 \in H(\mathbb{D})$, φ is an analytic self-map of $\mathbb{D}$ and n a positive integer.

Higher order mean value functions and polyharmonic functions

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We introduce integral mean value functions which are averages of integral means over spheres/balls and over their images under the action of a discrete group of complex rotations. In the case of real analytic functions we derive higher order Pizzetti's formulas. As applications we obtain a maximum principle for polyharmonic functions and a characterization of convergent solutions to higher order heat type equations. Also a new Dirichlet type problem for polyharmonic functions is introduced and solved in a special case.

On the structure of generalized analytic function in the vicinity of singular point

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The talk deals with the structure of the solutions of Carleman-Vekua equations in the neighborhood of singular point. The problem of the existence of a special majorant is studied. Some direct applications are presented. Talk is partially supported by the Shota Rustaveli National Science Foundation grant N 17-96

On the dimension of the kernel of a singular integral operator with non-Carleman shift and conjugation

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On the Hilbert space $\widetilde{L}_2(\mathbb{T})$ the singular integral operator with non-Carleman shift and conjugation $K = P_+ + (aI + AC)P_-$ is considered, where $P_\pm$ are the Cauchy projectors, $A = \sum_{j=0}^{m} a_j U^j$, a, a_j, $j = \overline{1,m}$, are continuous functions on the unit circle $\mathbb{T}$, U is the shift operator and C is the operator of complex conjugation. An estimate for the dimension of the kernel of the operator K is obtained; some particular cases are considered.

Global existence for coupled to the structurally damped σ-evolution model

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We are interested in the global existence to the coupled Cauchy problem for structurally damped σ-evolution models:

$$\begin{cases} u_{tt}(t,x) + (-\Delta)^\sigma u(t,x) + b(t)(-\Delta)^\delta u_t(t,x) = |v|^p, \\ u(0,x) := u_0(x), \quad u_t(0,x) := u_1(x) \\ \\ v_{tt}(t,x) + (-\Delta)^\sigma v(t,x) + \mu(-\Delta)^\delta v_t(t,x) = |u|^p, \\ v(0,x) := v_0(x), \quad v_t(0,x) := v_1(x), \end{cases}$$

for $(t,x) \in (0,\infty) \times \mathbb{R}^n$, where $\delta \in [0,\sigma]$, $\sigma > 1$ and $b = b(t)$ is a positive function.

On the existence and compactness of the resolvent of a Schrödinger operator with a negative parameter

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The talk is devoted to the questions of the existence of the resolvent and coercive estimates of the Schrödinger operator with a negative parameter in the space $L_2(R^n)$, where $n \geq 2$. We note that a negative Schrödinger operator with a negative parameter arises when studying singular differential operators of hyperbolic type in the space $L_2(R^{n+1})$.

To theory one class of three-dimensional integral equation with super-singular kernels by tube domain

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In this work, we investigate one class of three- dimensional integral equation by tube domains, are in power basis and lateral surface and may have supersingularity.

Let Ω denote the cylindrical domain $\Omega = \{(t,z) : a < t < b, |z| < R\}$. Lower ground this cylinder denote by $D = \{t = a, |z| < R\}$ and the lateral surface will be denote by $S = \{a < t < b, |z| = R\}, z = x + iy$. In domain Ω we shall consider the following integral equation

$$\varphi(t,z) + \int_a^t \frac{K_1(t,\tau)}{(\tau-a)^\alpha}\varphi(\tau,z)d\tau + \frac{1}{\pi}\iint\limits_D \frac{exp[i\theta]K_2(r,\rho)}{((R-\rho)^\beta)(s-z)}\varphi(t,s)ds + \frac{1}{\pi}\int_a^t \frac{d\tau}{(\tau-a)^\alpha}\iint\limits_D \frac{K_3(t,\tau;r,\rho)}{(R-\rho)^\beta(s-z)}exp[i\theta]\varphi(\tau,s)ds = f(t,z),$$

where $\theta = args, s = \xi + i\eta, ds = d\xi d\eta, \rho^2 = \xi^2 + \eta^2, r^2 = x^2 + y^2, K_1(t,\tau) = \sum_{j=1}^{n} A_j(\omega_a^\alpha(t) - \omega_a^\alpha(\tau))^{j-1}, K_2(r,\rho) = \sum_{l=1}^{m} B_l(\omega_a^\beta(r) - \omega_a^\beta(\rho))^{i-1}), K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho), A_j(1 \leq j \leq n), B_l(1 \leq l \leq m)$ -are given constants, $f(t,z)$-are given function, $\varphi(t,z)$-unknown function, $\omega_a^\alpha(t) = [(\alpha-1)(t-a)^{\alpha-1}]^{-1}$. In this work in depend of the roots of the characteristics equations

$$\lambda^n + \sum_{j=1}^{n} A_j(j-1)!\lambda^{n-j} = 0,$$

and

$$\mu^m + \sum_{l=1}^{m} B_j(j-1)!\mu^{n-j} = 0$$

obtained manifold solution. In this case, when general solution contain arbitrary functions, stand and investigation different boundary value problems.

2-D Frequency-Domain System Identification

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In this article, we propose two iterative algorithms to identify transfer functions of 2-D systems. The proposed algorithms are modifications of the two-dimensional Adaptive Fourier Decomposition (abbreviated as 2-D AFD) and Weak Pre-Orthogonal Adaptive Fourier Decomposition (abbreviated as W-POAFD). 2-D AFD and W-POAFD are newly established adaptive representation theories for multivariate functions utilizing, respectively, the product-TM system and the product-Szego dictionary. The proposed algorithms give rise to rational approximations with real coefficients to transfer functions. Owing to the modified maximal selection principles, the algorithms achieve a fast convergence rate $O(n^{-\frac{1}{2}})$. To use 2-D AFD and W-POAFD for system identification not only the theory is revised, but also the practical algorithm codes are provided. Experimental examples show that the proposed algorithms give promising results. The theory and algorithms studied in this paper are valid for any n-D case, $n \geq 2$.

Paley–Wiener-type Theorem for Analytic Functions in Tubular Domains

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Herein, a weighted version of the Paley–Wiener-type theorem for analytic functions in a tubular domain over a regular cone is obtained using H^p space method. Then, the classical n-dimensional Paley–Wiener theorem is generalized to a case wherein $0 < p < 2$ and K is not required to be a symmetric body. Finally, a version of the edge-of-the-wedge theorem is obtained as an application of the weighted theorems.

S.3 Complex Geometry

Organisers
Alexander Schmitt

Scope of the session: The main topics will be Higgs bundles and, more generally, augmented principal bundles. These are objects of complex algebraic geometry with links to other areas, such as arithmetic and mathematical physics. In the form of gauge theory, analysis plays a major role in the investigation of these objects. In fact, the so-called Kobayashi-Hitchin correspondence relates an algebro-geometric moduli space for these objects to a gauge theoretically defined one. The proof of the correspondence is highly analytical. The correspondence and its proof played a major role, e.g., in the work of Sir Simon Donaldson on invariants of differentiable four-manifolds for which he was awarded the fields medal.

—Abstracts—

Higgs bundles and Schottky representations

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We relate Schottky representations to certain Lagrangian subspaces of the moduli space of Higgs G-bundles (G is a connected reductive algebraic group). It is a fundamental result in the theory of Higgs bundles, the so-called non-abelian Hodge theorem, that by considering the Hitchin equations for G-Higgs bundless, one obtains a homeomorphism between the Betti space, B, and the moduli space of semistable G-Higgs bundles over a Riemann surface X. By a remark of Baraglia-Schaposnik, when considering G-Higgs bundles over X with a real structure, one is naturally lead to representations into G of the fundamental group of a 3-manifold with boundary X. These are naturally related to Schottky representations, as we will present in this talk. Our approach via Schottky representations has one advantage: we obtain a simple argument that shows that the Baraglia-Schaposnik brane is indeed Lagrangian with respect to the natural complex structure of B (coming from the complex structure of G). More precisely, we obtain a simpler proof of the vanishing of the complex symplectic form on the strict Schottky locus. This a joint work with Susana Ferreira (IPL) and Carlos Florentino (FCUL).

Intersection cohomology of the moduli space of Higgs bundles on a smooth projective curve

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Let X be a smooth projective curve of genus g over $\mathbb{C}$. The character variety $\mathcal{M}_B$ parametrizing conjugacy classes of representations from the fundamental group of X into $SL(2,\mathbb{C})$ is an affine irreducible singular projective variety. The Non Abelian Hodge theorem states that there is a real analytic isomorphism between $\mathcal{M}_B$ and the quasi projective singular variety $\mathcal{M}_{Dol}$ which parametrizes semistable Higgs bundles of rank 2 and degree 0 on X. During the seminar I will present a desingularization of these moduli spaces and I will compute the intersection cohomology of $\mathcal{M}_{Dol}$ using the famous Decomposition theorem by Beilinson, Bernstein, Deligne and Gabber. Moreover I will show that the mixed Hodge structure on the intersection cohomology is pure, showing evidence that an analogue of the $P = W$ conjecture might hold for singular moduli spaces.

Generating Functions for Hodge-Euler polynomials of GL(n,C)-character varieties

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Given a finitely generated group F and a complex reductive Lie group G, the G-character variety of F, $X_F G = Hom(F,G)/\!\!/G$, is typically a singular algebraic variety whose geometric and topological (and sometimes arithmetic) properties can be studied via mixed Hodge structures (MHS).
The most interesting cases are when F is the fundamental group of a Kähler manifold M, since then $X_F G$ is homeomorphic to a space of G-Higgs bundles over M. Some special classes of character varieties have their MHS encoded in a polynomial generalization of the Euler-Poincaré characteristic: the Hodge-Euler, also called the E-polynomial.
In this seminar, concentrating in the case of the general linear group $G = GL(n,\mathbb{C})$, we present a remarkable relation between the E-polynomials of $X_F G$ and those of $X_F^{irr} G$, the locus of *irreducible representations* of F into G. We will also give an overview of known explicit computations of E-polynomials, as well as some conjectures and open problems.

Cartan branes on the Hitchin system

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We study mirror symmetry on the singular locus of the Hitchin system at two levels. Firstly, by covering it by (supports of) BBB-branes, corresponding to Higgs bundles reducing their structure group to the Levi subgroup of some parabolic subgroup P, whose conjectural dual BAA-branes we identify. Heuristically speaking, the latter are given by Higgs bundles reducing their structure group to the unipotent radical of P. Secondly, when P is a Borel subgroup, we are able to construct a family of hyperholomorphic bundles on the BBB-brane, and study the variation of the dual under this choice. We give evidence of both families of branes being dual under mirror symmetry via an ad-hoc Fourier-Mukai integral functor.

$SO(p,q)-$Higgs bundles and higher Teichmüller components

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Some connected components of a moduli space are mundane in the sense that they are distinguished only by obvious topological invariants or have no special characteristics. Others, such as the Hitchin component in the moduli space of Higgs bundles, are more alluring and unusual either because they are not detected by primary invariants, or because they have special geometric significance, or both. In this paper we describe new examples of such "exotic" components in moduli spaces of $SO(p,q)-$Higgs bundles on closed Riemann surfaces or, equivalently, moduli spaces of surface group representations into the Lie group $SO(p,q)$. We also provide a complete count of the connected components of these moduli spaces (except for $SO(2,q)$, with $q>3$). Time permitting, we will comment on possible generalizations. The talk will mainly be based on arXiv:1802.08093 which is joint work with Marta Aparicio-Arroyo, Steven Bradlow, Brian Collier, Oscar García-Prada and André Oliveira.

The Riemann-Hilbert mapping in genus two

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One possible formulation of the Riemann-Hilbert problem in higher genus is ask which is the vector bundle underlying the holomorphic connection over a curve associated to a given monodromy representation. Since the monodromy is given in terms of the topological and not the complex structure of the curve, one may vary the latter and obtains, by Riemann-Hilbert correspondence, an isomonodromic family of connections. In collaboration with F. Loray, we obtained the following result: in the moduli space of irreducible $\mathfrak{sl}_2\mathbb{C}$-connections over genus two curves, the isomonodromic foliation is transversal to the locus of the trivial bundle and transversal to the locus of flat unstable bundles. In this talk, we will present some applications of this result and of its proof.

Universal torsors over degenerating del Pezzo surfaces

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Let S be a split one-parameter family of smooth del Pezzo surfaces degenerating to a del Pezzo surface with **ADE**-singularities. Let G be the reductive group given by the root system of these singularities. We construct a G-torsor (or principal G-bundle) over S whose restriction to the smooth fibres is the extension of structure group of the universal torsor under the Néron-Severi torus introduced and studied by Colliot-Thélène and Sansuc. The G-torsor is unique and infinitesimally rigid. This extends a construction of Friedman and Morgan for individual singular del Pezzo surfaces. It is joint work with Ulrich Derenthal.

Compactification of the moduli space of stable principal G-bundles over a stable curve and beyond

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Given a projective manifold X, a reductive group G and a faithfull representation $\rho: G \to \mathrm{SL}(V)$, A. Schmitt defined a singular principal G-bundle on it as a pair (F,τ) given by a torsion free sheaf and a morphism of algebras $\tau: S^\bullet(V\otimes F)^G \to O_X$, and proved the existence of a compact moduli space for δ-(semi)stable singular principal G-bundles having the space of stable principal G-bundles as an open subscheme. U. Bhosle proved the existence of a compact moduli space for δ-(semi)stable singular principal G-bundles over irreducible projective curves with at most nodes as singularities. An important feature of this moduli space is that, when the curve is smooth, it is isomorphic to the classical moduli space constructed by A. Ramanathan provided the rational parameter δ is large enough. When the base curve is reducible, we can generalize the definition of singular principal bundle by considering sheaves of depth one. In this talk, I will discuss the existence of a compact moduli space for δ-(semi)stable singular principal G-bundles over nodal (possibly reducible) projective curves and its behavior under variations of the base curve along $\overline{\mathcal{M}_g}$.

Klein coverings of genus 2 curves

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We consider étale 4 : 1 coverings of smooth genus 2 curves with the monodromy group the Klein group. Depending on the values of the Weil pairing restricted to the group defining the covering, we distinguish the isotropic and non-isotropic case. In this talk we will discuss the correspondence between the non-isotropic Klein coverings and the $(1,4)$-polarised abelian surface. As a consequence of this, one can show the existence

of exactly four hyperelliptic curves in a general $(1,4)$-polarised abelian surface. We will also give several characterisations of the Klein coverings (isotropic and non-isotropic) leading to the result that the corresponding Prym maps are generically injective in both cases.
This is a joint work with Paweł Borówka.

Mirror symmetry on some non generic loci of the Hitchin system

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Mirror symmetry for Hitchin systems has been proven to be realised by a relative Fourier-Mukai transform over the generic locus. In this talk I will explain how mirror symmetry operates on some natural hyperholomorphic branes on the Hitchin system, given by fixed points by tensorisation with a torsion line bundle. Generically, they are supported over the locus of singular integral spectral curves. Many aspects of their geometry are however more easily understood through branes supported on the most singular locus of a related Hitchin system. I will refer to this interplay during the exposition.
This is joint work with E. Franco, P. Gothen and A. Oliveira.

Surface braid groups, finite Heisenberg covers and double Kodaira fibrations

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A *Kodaira fibration* is a smooth, connected holomorphic fibration $f\colon S \to B$, where S is a compact complex surface and B is a compact complex curve, which is not isotrivial (this means that not all its fibres are biholomorphic to each other). Examples of such fibrations were originally constructed by Kodaira as a way to show that, unlike the topological Euler characteristic, the signature σ of a manifold is not multiplicative for fibre bundles. In fact, every Kodaira fibred surface S satisfies $\sigma(S) > 0$, whereas $\sigma(B) = \sigma(F) = 0$, and so $\sigma(S) \neq \sigma(B)\sigma(F)$. On the other hand, in a classical work by Chern, Hirzebruch and Serre it is proved that the signature is multiplicative for differentiable fibre bundles in the case where the monodromy action of the fundamental group $\pi_1(B)$ on the rational cohomology ring $H^*(F, \mathbb{Q})$ is trivial; thus, Kodaira fibrations provide examples of fibre bundles for which this action is non-trivial. In this talk, we show how to construct new examples of double Kodaira fibrations by using finite Heisenberg covers (i.e., Galois covers with Galois group isomorphic to a finite Heisenberg group) of a product $\Sigma_b \times \Sigma_b$, where Σ_b is a smooth projective curve of genus $b \geq 2$. Each cover is obtained by providing an explicit group epimorphism from the pure braid group $\mathsf{P}_2(\Sigma_b)$ to the corresponding Heisenberg group. In particular, we exhibit the first examples of surface that admits two distinct Kodaira fibrations with base genus 2, answering a stronger version of a problem from Kirby's list in low-dimensional topology:

Theorem. *There exists an oriented* 4*-manifold* X *of signature* 144 *that can be realized as a surface bundle over a surface of genus* 2 *with fibre genus* 325 *in two different ways.*

This is joint work with A. Causin.

On Galois group of factorized covers of curves

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Let $\mathcal{Y} \xrightarrow{\psi} \mathcal{X} \xrightarrow{\varphi} \mathbb{P}^1$ be a sequence of covers of compact Riemann surfaces. In this work we study and completely characterize the Galois group $\mathfrak{G}(\varphi\circ\psi)$ of $\varphi\circ\psi$ under the following properties: φ is a simple cover of degree m and ψ is a Galois unramified cover of degree n with abelian Galois group of type $(n_1, n_2, ...n_s)$.
We prove that $\mathfrak{G}(\varphi\circ\psi) \cong (\mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2} \times \cdots \mathbb{Z}_{n_s})^{m-1} \rtimes \mathbf{S}_m$. Furthermore, we give a natural geometric generator system of $\mathfrak{G}(\varphi\circ\psi)$ obtained by studying the action on the compact Riemann surface $\mathcal{Z}$ associated to the Galois closure of $\varphi \circ \psi$.

Higher Teichmüller spaces for orbifolds

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The Teichmüller space of a compact 2-orbifold X can be defined as the space of faithful and discrete representations of the fundamental group of X into $\mathbf{PGL}(2,\mathbb{R})$. It is a contractible space. For closed orientable surfaces, "Higher analogues" of the Teichmüller space are, by definition, (unions of) connected components of representation varieties of $\pi_1(X)$ that consist entirely of discrete and faithful representations. There are two known families of such spaces, namely Hitchin representations and maximal representations, and conjectures on how to find others. In joint work with Daniele Alessandrini and Gye-Seon Lee, we show that the natural generalisation of Hitchin components to the orbifold case yield new examples of Higher Teichmüller spaces: Hitchin representations of orbifold fundamental groups are discrete and faithful, and share many other properties of Hitchin representations of surface groups. However, we also uncover new phenomena, which are specific to the orbifold case.

Theta functions on moduli spaces of local systems

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Moduli spaces of SL(2,$\mathbb{C}$)-local systems on punctured Riemann surfaces provided examples of log Calabi-Yau varieties, for which a generalisation of the classical theta functions has been developed by Gross and Siebert. We

will see an explicit computation and a modular interpretation of these theta functions for the moduli space of local systems of the four-punctured sphere, which is the total space of a family of affine cubic surfaces. Further we will discuss the relationship with the complex geometry of a diffeomorphic family of rational elliptic surfaces with a singular fibre of type I_0^*, the complement of which is the corresponding moduli space of Higgs bundles.

S.4 Complex Variables and Potential Theory

Organisers
Tahir Aliyev Azeroglu,
Massimo Lanza de Cristoforis,
Anatoly Golberg, Sergiy Plaksa

Scope of the session: This session is devoted to the wide range of directions of complex analysis, potential theory, their applications and related topics.

—Abstracts—

Some Remarks on the Uniqueness Part of Schwarz Lemma

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Let f be an holomorphic function from the unit disc to itself. We generalize Schwarz Lemma at the boundary improving the condition on the bilogarithmic concave majorant and provide conditions on the local behaviour of f along boundary near a finite set of the boundary points that requires f to be a finite Blaschke product. The basis of proofs of the main results is based on a special version of Phragmen-Lindelöf Princible.
KeyWords. Holomorphic function, Harnack inequality, Bilogarithmic concave majorant, Phragmen-Lindelöf princible.

Stokes' Theorem and the Cauchy - Pompeiu Formula for Polydisc

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The talk will present some challenges in finding the Cauchy - Pompeiu formula for polydiscs. It is shown by Hormander and others that the Stokes' Theorem is the key in establishing the Cauchy - Pompeiu formula for general functions on a disc in one dimensional complex domain. However the Stokes' Theorem for polydiscs is not readily available and its proof is an open problem. For this purpose, differential forms could be applied as it was done in one variable case and in ball like domains. Questions remain for the polydisc: the Stokes' Theorem is applicable to which boundary? To the characteristic boundary or to the entire boundary of the polydisc? The talk articulate on these points.

Completeness theorems for systems of particular solutions of partial differential equations

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Roughly speaking there are two different kinds of completeness theorems for systems of particular solutions of partial differential equations. Results of a first kind show that we can approximate in a domain a solution of a partial differential equation by a sequence of particular solutions of the same equation. For example, if we have a holomorphic function f of one complex variable, we may ask when f can be approximated in some norms by polynomials or by rational functions. The classical theorems of Runge and Mergelyan are the main results in this direction. These problems have been widely studied and extended to general elliptic partial differential equations. Results of a second kind are the so called completeness theorems in the sense of Picone. They are much more sophisticated and related not only to a partial differential equation, but also to a particular boundary value problem. The present talk is devoted to consider the problem of completeness in this last sense. In particular we shall give necessary and sufficient conditions for the completeness of polynomial solutions of a partial differential equations of higher order related to the Dirichlet problem. We shall give also some recent results obtained for systems, where very little is known. All these results have been obtained by using potential theory.

Existence, uniqueness, and regularity properties of the solutions of a nonlinear transmission problem

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We consider a nonlinear contact problem that arises in the study of composite structures glued together by thermo-active materials. By an approach based on boundary integral equations and on the Schauder fixed-point theorem we can prove that the problem has solutions; however, such solution may not be locally unique and may be also very irregular. For example, we might have solutions that are not in H^s for any $s > 1/2$. The results presented are obtained in collaboration with B. Luczak (the University of Tulsa, US), G. Mishuris (Aberystwyth University, UK), R. Molinarolo (Aberystwyth University, UK), and P. Musolino (Università degli Studi di Padova, Italy).

Extremal decomposition of the complex plane with free poles

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In geometric function theory of complex variable extremal problems on non-overlapping domains are well-known classic direction. A lot of such problems are reduced to determination of the maximum of product of inner radii on the system of non-overlapping domains satisfying a certain conditions. We consider the well-known problem of maximum of the functional

$$r^{\gamma}(B_0,0)\prod_{k=1}^{n} r(B_k,a_k),$$

where B_0,...,B_n ($n \geqslant 2$) are pairwise disjoint domains in $\overline{\mathbb{C}}$, $a_0 = 0$, $|a_k| = 1$, $k = \overline{1,n}$ are different points of the unit circle, $\gamma \in (0,n]$, and $r(B,a)$ is the inner radius of the domain $B \subset \overline{\mathbb{C}}$ relative to a point a. This problem was posed as an open problem in the Dubinin paper in 1994. Till now, the problem has not been solved, though some partial solutions are available. The proof is due to Dubinin for $\gamma = 1$ and to Kuz'mina for $0 < \gamma < 1$. Subsequently, Kovalev in 1996 solved this problem for $n \geqslant 5$ under the additional assumption that the angles between neighbouring line segments $[0,a_k]$ do not exceed $2\pi/\sqrt{\gamma}$. We obtained evaluation of the functional for all n and $\gamma \in (1,n]$. **Theorem.** *Let $n \in \mathbb{N}$, $n \geqslant 2$, $\gamma \in (1,\, n]$. Then, for any system of different points $\{a_k\}_{k=1}^{n}$ of the unit circle and any mutually non-overlapping domains B_k, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{0,n}$, $a_0 = 0$, the following inequality holds*

$$r^{\gamma}(B_0,0)\prod_{k=1}^{n} r(B_k,a_k) \leqslant n^{-\frac{\gamma}{2}}\left(\frac{4}{n}\right)^{n-\gamma}.$$

On the irregular generalized Cauchy-Riemann equations

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This is a joint work with V.Jikia and G.Makatsaria (Singular Generalized Analytic Functions J.Math. Sci. 237, 30-109, 2019). We consider the generalized Cauchy-Riemann equation (*Carleman-Bers-Vekua equation* in complex form)

$$\frac{\partial w}{\partial \overline{z}} = aw + b\overline{w},$$

where $a \in L_p^{loc}(\mathbb{C})$ and $b \in L_{p,2}(\mathbb{C}), p > 2$. Introduce subclasses of the class $L_p^{loc}(\mathbb{C})$, $p > 2$, elements of which have $\dfrac{\partial}{\partial \overline{z}}$ primitives and satisfying certain additional asymptotic conditions. In the talk we define these classes and prove their properties. In particular, denote by $J_0(\mathbb{C})$ the set of functions $a \in L_p^{loc}(\mathbb{C})$, $p > 2$ for which there exists $\dfrac{\partial}{\partial \overline{z}}$-primitive $Q(z)$ satisfying the following condition

$$ReQ(z) = O(1), \quad z \in \mathbb{C}$$

and denote by $J_1(\mathbb{C})$ the set of the functions $a \in L_p^{loc}(\mathbb{C})$, $p > 2$, for which there exists $\dfrac{\partial}{\partial \overline{z}}$ primitive $Q(z)$, satisfying the following condition

$$z^n \exp\{Q(z)\} = O(1), \quad z \in \mathbb{C},$$

for every natural number n.

Theorem. *The function $a(z)$ of the class $L_p^{loc}(\mathbb{C})$, $p > 2$, belongs to the class $J_1(\mathbb{C})$ if and only if its $\dfrac{\partial}{\partial \overline{z}}$-primitive exists and satisfies the condition*

$$\lim_{z\to\infty} z^k \exp\{Q(z)\} = 0,$$

for every natural number k.

Theorem. *The class $J_0(\mathbb{C})$ is a linear space over the field of real numbers. Moreover, for arbitrary real $p > 2$ the following inclusion*

$$L_{p,2}(\mathbb{C}) \subset J_0(\mathbb{C})$$

holds.

Theorem. *The following equality*

$$J_0(\mathbb{C}) \cap J_1() = \varnothing$$

holds.

This work was supported, in part, by the Shota Rustaveli National Science Foundation under Grant No 17-96.

Nonlinear Beltrami equation

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We consider a nonlinear counterpart of the classical Beltrami equation and study the main features of its solutions. It involves directional dilatations connected with a priory fixed point and a class of mappings called ring Q-homeomorphisms with respect to p-module. We also establish some regularity properties of solutions to such equation and illustrate them by several examples.

Harmonic measure distribution functions on spherical and toroidal surfaces

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It will be shown how to generalise recent formulae for harmonic measure distribution functions, or h-functions, for multiply connected slit domains in the complex plane to two distinct compact surfaces: the sphere (genus-0) and the ring torus (genus-1).
Given a domain Ω on a compact surface $\mathcal{S}$, and a fixed basepoint $z_0 \in \Omega$, the h-function is a piecewise smooth continuous function $h : [0,\infty) \to [0,1]$ which encodes certain properties of the triple $(\mathcal{S},\Omega,z_0)$. For $r > 0$, the value of $h(r)$ is the harmonic measure of the portion of the boundary $\partial\Omega$ that lies within a distance r of z_0, and where r is measured along the surface $\mathcal{S}$.
Motivated by deriving analytical formulae, attention is restricted to domains Ω exterior to a finite number of horizontal slits of equal latitude, and the associated h-functions determined using a combination of techniques from conformal mapping and special function theory. The formulae derived hold for any finite number of slits.

Monogenic functions in 2-D commutative Complex algebras to plane orthotropy

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The present research is devoted to the construction of classes of "analytic" functions Φ with values in two-dimensional commutative algebras over the field of complex numbers containing bases (e_1, e_2) with some algebraic properties (in what follows, we construct mentioned bases and the corresponding algebra in the explicit form) sufficient for the real components of these functions to satisfy the following equation:

$$(1) \qquad \left(\frac{\partial^4}{\partial y^4} + 2p\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial x^4}\right) u(x,y) = 0,$$

where $p > -1$ (a case of elliptic type), u is a real-valued solution of (**??**), an argument $(x, y) \in D$, while the latter is belonging to the Cartesian plane xOy. Eq. (1)for $p \neq 1$ is an equation of the stress function of some cass of orthotropic plane deformations, an appropriate algebra is semi-simple. The case $p = 1$ is corresponding to isotropic plane deformations, Eq. (1) terns onto the biharmonic Eq., an appropriate algebra is the biharmonic (cf., e.g., papers of S. G. Gryshchuk, S. A. Plaksa and I. P. Mel'nichenko). A characterisation of solutions u is done by means of components of $\Phi(xe_1 + ye_2)$, $(x, y) \in D$, in case when a domain under consideration D is bounded and simply connected. A solution of an appropriate equilibrium system in displacements is found as a linear combinations of all real-valued components of monogenic functions $\Phi(xe_1 + ye_2)$.

Linearization of holomorphic semicocycles

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Semicocycles over semigroups of holomorphic self-mappings appear naturally in the study of non-autonomous dynamical systems in Banach spaces. In the theory of semicocycles over semigroups, a basic relation is the cohomological equivalence. Namely, two cohomological semicocycles have similar properties including continuity and differentiability, asymptotic behavior, and so on. So that, one central goal in study of semicocycles is to classify them up to relation of cohomology. In particular, the linearization problem is that of determining whether a given semicocycle is cohomologous to a constant one (that is, independent of the space-variables). Focusing on this problem, we provide some criteria for a semicocycle to be linearizable. These conditions are essential even for semicocycles over linear semigroups. **This talk is based on a joint work with M. Elin and G. Katriel**

Lower bounds for the volume of the image of a ball

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Let Γ be a family of curves γ in $\mathbb{R}^n$, $n \geqslant 2$. A Borel measurable function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, (abbr. $\rho \in \operatorname{adm}\Gamma$), if

$$\int\limits_{\gamma} \rho(x)\, ds \;\geqslant\; 1$$

for any curve $\gamma \in \Gamma$. Let $p \in (1, \infty)$. The quantity

$$M_p(\Gamma) \;=\; \inf_{\rho \in \operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x)\, dm(x)\,.$$

is called *p–modulus* of the family Γ. For arbitrary sets E, F and G of $\mathbb{R}^n$ we denote by $\Delta(E, F, G)$ a set of all continuous curves $\gamma : [a, b] \to \mathbb{R}^n$, that connect E and F in G, i. e., such that $\gamma(a) \in E$, $\gamma(b) \in F$ and $\gamma(t) \in G$ for $a < t < b$. Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$, $x_0 \in D$ and $d_0 = dist(x_0, \partial D)$. Set

$$\mathbb{A}(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}\,,$$

$$S_i = S(x_0, r_i) = \{x \in \mathbb{R}^n : |x - x_0| = r_i\}\,, \quad i = 1, 2\,.$$

Let a function $Q : D \to [0, \infty]$ be Lebesgue measurable. We say that a homeomorphism $f : D \to \mathbb{R}^n$ is ring Q-homeomorphism with respect to p-modulus at $x_0 \in D$, if the relation

$$M_p(\Delta(fS_1, fS_2, fD)) \;\leqslant\; \int\limits_{\mathbb{A}} Q(x)\, \eta^p(|x - x_0|)\, dm(x)$$

holds for any ring $\mathbb{A} = \mathbb{A}(x_0, r_1, r_2)$, $0 < r_1 < r_2 < d_0$, $d_0 = dist(x_0, \partial D)$, and for any measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that

$$\int\limits_{r_1}^{r_2} \eta(r)\, dr = 1\,.$$

Theorem. *Let D be a bounded domain in $\mathbb{R}^n$, $n \geqslant 2$, and $f : D \to \mathbb{R}^n$ be a ring Q-homeomorphism with respect to p-modulus at the point $x_0 \in D$ for $p > n$. Assume that the function Q satisfies the condition*

$$q_{x_0}(t) \leqslant q_0\, t^{-\alpha},\; q_0 \in (0, \infty)\,,\; \alpha \in (0, \infty)\,,$$

for $x_0 \in D$ and almost all $t \in (0, \varepsilon_0)$, $\varepsilon_0 \in (0, d_0)$, $d_0 = dist(x_0, \partial D)$. Then for all $r \in (0, \varepsilon_0)$ the estimate

$$m(fB(x_0, r)) \geqslant \Omega_n \left(\frac{p-n}{\alpha + p - n}\right)^{\frac{n(p-1)}{p-n}} q_0^{\frac{n}{n-p}}\, r^{\frac{n(\alpha+p-n)}{p-n}}$$

holds true, where $B(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| \leqslant r\}$, $q_{x_0}(t) = \frac{1}{\omega_{n-1}\, r^{n-1}} \int\limits_{S(x_0, r)} Q(x)\, d\mathcal{A}$ is the integral mean value over the sphere $S(x_0, t) = \{x \in \mathbb{R}^n : |x - x_0| = t\}$, Ω_n is the volume of the unit ball in $\mathbb{R}^n$, ω_{n-1} is the surface area of the unit sphere $\mathbb{S}^{n-1}$ in $\mathbb{R}^n$, $d\mathcal{A}$ is the element of the surface area.

Approximation properties and related results for univalent mappings in higher dimensions

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Let $n \geq 2$ and let $A \in L(\mathbb{C}^n)$ be such that $\Re\langle A(z), z\rangle > 0$, $\|z\| = 1$. Also, let $S_A^0(\mathbb{B}^n)$ be the family of normalized univalent mappings on $\mathbb{B}^n$ which have A-parametric representation. In this talk we present a variational method for A-normalized univalent subordination chains on the Euclidean unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$, which allows us to obtain approximation properties for the family $S_A^0(\mathbb{B}^n)$ by automorphisms of $\mathbb{C}^n$. In particular, we present approximation results of starlike, convex, and spirallike mappings by automorphisms of $\mathbb{C}^n$ whose restrictions to $\mathbb{B}^n$ have the same geometric property. Extremal properties for the family $S_A^0(\mathbb{B}^n)$ will be also discussed. Finally, certain questions and open problems will be mentioned.

An inequality for Hölder continuous functions, in the wake of the work of Carlo Miranda

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We plan to show an inequality for Hölder continuous functions, which is useful to study the boundary behavior of layer potentials and which enables to simplify a proof of a result of Carlo Miranda.

Shape analysis of the longitudinal flow along a periodic array of cylinders

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In the present talk we study the behavior of the longitudinal flow along a periodic array of cylinders upon perturbations of the shape of the cross section of the cylinders and the periodicity structure, when a Newtonian fluid is flowing at low Reynolds numbers around the cylinders. The periodicity cell is a rectangle of sides of length l and $1/l$, where l is a positive parameter, and the shape of the cross section of the cylinders is determined by the image of a base domain through a diffeomorphism ϕ. We also assume that the pressure gradient is parallel to the cylinders. Under such assumptions, for each pair (l, ϕ), one defines the average of the longitudinal component of the flow velocity $\Sigma[l, \phi]$. As the main result, we show that the quantity $\Sigma[l, \phi]$ depends analytically on the pair (l, ϕ), which we consider as a point in a suitable Banach space.

The results presented have been obtained in collaboration with Paolo Musolino and Roman Pukhtaievych.

On polynomial extension property

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The celebrated theorem of H. Cartan states that for an analytic variety V in a pseudoconvex domain Ω, any holomorphic function on V extends to a holomorphic function on Ω. In this talk we are interested in these subsets V that additionally admit the (polynomial) extension property, meaning that for every bounded holomorphic function (resp. polynomial) f on V there exists a bounded holomorphic function F on Ω such that

$$F|_V = f \quad \text{and} \quad \sup_{\Omega} |F| = \sup_{V} |f|.$$

The characterization of subsets in the bidisc, that have the polynomial extension property, were achieved by Agler and M^{c}Carthy (2003). More recently Kosiński and M^{c}Carthy (2017) were studied one- and two-dimensional subsets of the tridisc. Based on the techniques from the latter, we have obtained the result, that an one-dimensional algebraic subset of arbitrarily dimensional polydisc $\mathbb{D}^n$, which has the polynomial extension property, is a holomorphic retract.

Converging expansions for Lipschitz self-similar perforations of a plane sector

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In contrast with the well-known methods of matching asymptotics and multiscale (or compound) asymptotics, the "Functional Analytic Approach" proposed by Lanza de Cristoforis allows to prove convergence of expansions around interior small holes of size ε for solutions of elliptic boundary value problems. Using the method of layer potentials, the asymptotic behavior of the solution as ε tends to zero is described not only by asymptotic series in powers of ε, but by convergent power series. In this talk we present some recent results, where we use this method to investigate the Dirichlet problem for the Laplace operator where holes are collapsing at a polygonal corner of opening ω. The strategy relies on a combination of odd reflections and conformal mappings so that the original problem is transformed into a similar problem where the perforations are near the center of a disc, on potential theory on Lipschitz domains, and on a detailed analysis of the solution of the limiting problem in proximity of the corner. We show that in addition to the scale ε there appears the scale $\eta = \varepsilon^{\pi/\omega}$ when π/ω is irrational, the solution of the Dirichlet problem is given by convergent series in powers of these two small parameters. The final outcome can be compared with multi-scale expansions for which convergence does not hold in general.
Based on joint work with M. Costabel, M. Dalla Riva, and M. Dauge.

Numerical computation of conformal capacity of generalized condensers

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Consider generalized condensers of the form $C = (\overline{\mathbb{C}}, E, \delta)$ where $E = \{E_k\}_{k=1}^m$ is a collection of nonempty closed pairwise disjoint sets in $\overline{\mathbb{C}}$ and $\delta = \{\delta_k\}_{k=1}^m$ is a collection of real numbers containing at least two different numbers. We assume that $G = \overline{\mathbb{C}} \setminus \cup_{k=1}^m E_k$ is a multiply connected domain of connectivity m such that $\partial G = \cup_{k=1}^m \partial E_k$ has no isolated boundary points. The set G is called the field of the condenser C, the sets E_k are the plates of the condenser, and the δ_k are the levels of the potential of the plates E_k, $k = 1, 2, \ldots, m$. The conformal capacity of C, $\mathrm{cap}(C)$, is then given by the Dirichlet integral

$$\mathrm{cap}(C) = \iint\limits_G |\nabla u|^2 dxdy$$

where u is the potential function of the condenser C, i.e., u is continuous in $\overline{G}$, harmonic in G, and equal to δ_k on ∂E_k for $k = 1, 2, \ldots, m$.
We present a boundary integral method for numerical computation of the capacity $\mathrm{cap}(C)$. The method is based on the boundary integral equation with the generalized Neumann kernel. The presented method applies to a wide variety of generalized condenser geometry including the cases when the plates of the generalized condenser are smooth Jordan curves, piecewise smooth Jordan curves, segment slits and circular slits.

The talk is based on collaborations with Professor Matti Vuorinen (University of Turku, Finland; vuorinen@utu.fi).

Support of Borel measures in the plane satisfying a certain positivity condition

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Certain Borel measures in the plane appear as representing parameters for classes of holomorphic functions with non-negative real or imaginary part. Such measures must satisfy a positivity condition which constitutes a severe restriction within the set of Borel measures. In this talk, we will present how the positivity condition that characterizes representing measures of holomorphic functions of two variables with non-negative imaginary part restricts the support of these measures. For example, we will present some subsets of the plane that cannot contain the support of such measures. This talk is based on joint work with Annemarie Luger.

Nonlocal boundary value problems for the Laplace operator in a unit ball which are multidimensional generalizations of the Samarskii-Ionkin problem

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In this talk we consider one stationary diffusion problem describing the Poisson equation. The problem is considered in a model domain, chosen as a half disk. Dirichlet classical boundary conditions are set on the arc of the circle. New nonlocal boundary conditions are set on the bottom base. The first condition means an equality of flows through opposite radii, and the second condition is the proportionality of distribution densities on these radii with a variable coefficient of proportionality. The uniqueness and existence of a classical solution to the problem are proved. An inverse problem on the solution definition to the Poisson equation and its right-hand part depending only on an angular variable are considered. As an additional condition we use the boundary overdetermination. Inverse problems to the Dirichlet, Neumann problems and to problems with nonlocal conditions of the equality of flows through the opposite radii are considered. The well-posedness of the formulated inverse problems is proved. Some of our results can be found in [1]. *The talk is based on joint work with Aishabibi Dukenbayeva (al-Farabi Kazakh National University and Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan).*
[1] M.A. Sadybekov and A.A. Dukenbayeva, *Direct and inverse problems for the Poisson equation with equality of flows on a part of the boundary*, Complex Variables and Elliptic Equations, **64**(5) (2019), 777-791, DOI: 10.1080/17476933.2018.1517340

Hypercomplex method for solving linear PDEs

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Algebraic-analytic approach to constructing solutions for given linear partial differential equations were investigated in many papers. It involves solving two problems. Problem (P 1) is to describe all the sets of vectors $e_1, e_2, \ldots, e_d$ of commutative associative algebra, which satisfy the polynomial characteristic equation (or specify the procedure by which they can be found). And the Problem (P 2) is to describe all the components of monogenic (i. e., continuous and differentiable in sense Gateaux) functions. Earlier we got a constructive description of all analytic functions with values is obtained in an arbitrary finite-dimensional commutative associative algebra over the field $\mathbb{C}$. Further we states that it is enough to limit the study of monogenic functions in algebras with the basis of $\{1, \eta_1, \eta_2, \ldots, \eta_{n-1}\}$, where $\eta_1, \eta_2, \ldots, \eta_{n-1}$ are nilpotents. In addition, it is showed that in each algebra with a basis of the form $\{1, \eta_1, \eta_2, \ldots, \eta_{n-1}\}$ the polynomial characteristic equation has solutions. That is, the problems (P 1) and (P 2) are completely solved on the classes of commutative associative algebras with the basis $\{1, \eta_1, \eta_2, \ldots, \eta_{n-1}\}$. It is worth noting that in a finite-dimensional algebra a decomposition of monogenic functions has a finite number of components, and therefore, it generates a finite number of solutions of a given partial differential equations. In this report we propose a procedure for constructing an infinite number of families of solutions of given linear differential equations with partial derivatives with constant coefficients. We use monogenic functions that are defined on some sequences of commutative associative algebras over the field of complex numbers. To achieve this goal, we first study the solutions of the so-called characteristic equation on a given sequence of algebras. Further, we investigate monogenic functions on

the sequence of algebras and study their relation with solutions of partial deferential equations. The proposed method is used to construct solutions of some equations of mathematical physics. In particular, for the three-dimensional Laplace equation and the wave equation, for the equation of transverse oscillations of the elastic rod and the conjugate equation, a generalized biharmonic equation and the two-dimensional Helmholtz equation. We note that this method yields all analytic solutions of the two-dimensional Laplace equation and the two-dimensional biharmonic equation (Goursat formula).

Weierstrass Theorem for Bicomplex Holomorphic Functions

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How are the zeros of a bicomplex holomorphic function? So, how is the corresponding Weierstrass theorem to prescribe zeros and singularities of a bicomplex function? In this talk we present differences between the classical Weierstrass theorem for analytic functions and the corresponding statement for bicomplex functions.

S.5 Constructive Methods in the Theory of Composite and Porous Media

Organisers
Vladimir Mityushev

Scope of the session: The topic concerns application of mathematical methods to study composite and porous media. In particular, it includes

- Effective properties of composites
- Homogenization theory
- Electromagnetic and optical properties of metamaterials
- Transport in random media
- Nanocomposites
- Cloaking

The theoretical methods applied in this field are usually based on complex analysis, boundary value problems, partial differential equations, functional equations etc. This session will be also interesting for engineers and designers using mathematical models and computer simulations. It is plan to organize a workshop within the session concerning mathematical models and applied problems presented by engineers.

—Abstracts—

Fast method for 2D Dirichlet problem for circular multiply connected domains

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In this paper we present the implementation of algorithm to determine the conductivity of 2D plane with non-overlapping disk inclusions. The fast Poincaré series method and modified Delaunay triangulation to determine closest neighbours was used.

Local Fields

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Consider a plane multiply connected domain D bounded by non-overlapping disks. Introduce the complex potential $u(z) = \mathrm{Re}\ \varphi(z)$ in D where the function $\varphi(z)$ is analytic in D except at infinity where $\varphi(z) \sim z$. The unknown function $\varphi(z)$ is continuously differentiable in the closures of the considered domain. We solve approximately the Schwarz problem when $u(z) = \mathrm{Re}\ \varphi(z)$ is equal to an undetermined constant on every boundary component of D by a method of functional equations

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The Robin problem in conical domains

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We study the existence and behavior near a boundary angular or conical point of weak solutions to the Robin problem for an elliptic quasi-linear second-order equation with the $p-$ and variable $p(x)$-Laplacian. Co-author of talk: Mariusz Bodzioch, University of Warmia and Mazury in Olsztyn, Poland

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Conductivity of two-dimensional composites with randomly distributed elliptical inclusions: Random elliptical inclusions

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Analytical approximate formulae for the effective conductivity tensor of the two dimensional composites with elliptical inclusions are derived in the form of polynomial approximations in concentration. New formulae explain the seeming contradiction between various formulae derived in the framework of self consistent methods. Random composites with high conducting inclusions of two different shapes (elliptical and circle) of the

same area are compared. It is established that greater relative concentration of ellipses increases the effective conductivity.

Estimations of the effective conductivity of composites with non-circular inclusions

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The closed form formula for the efective conductivity of 2D composite material with nonoverlapping ircular inclusions is known. We present a method and results on estimations of the efective conductivity of omposites with non-circular inclusions by approximating their geometry, i.e. by covering the considered shape with disks which form cluster. In addition to a general discussion of the problem of approximation, in this talk, we present an estimation of the effective conductivity of composite materials with triangular inclusions.

Effective elastic constants for 2D random composites

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Basing on the MMM principle by Hashin and the theory of homogenization we follow the method of random constructive homogenization developed in Gluzman S., Mityushev V., Nawalaniec W., (2018). Consider 2D multiphase random composites with different radii circular inclusions located at the sites of hexagonal (triangular) lattice. The inclusions are embedded into the matrix with different elastic properties. Plane strain elastic problem is solved for such a composite. The effective shear and bulk moduli are obtained in the form of power series in the inclusions concentration f. The coefficients of this series are written in analytical form, with the coefficients expressed through the elastic constants of components. New analytical formulae for the effective constants are deduced up to arbitrary $O(f^n)$ for macroscopically isotropic composites. We derive general analytical formulae for the local fields and for the effective constants in 2D random composites. We consider examples of simulated random media and application of the derived symbolic-numerical algorithms to them.

Education of Selected Notions Connected with Mathematical Analysis with Help of Visualization

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The aspect of visualization plays an important role in the analysis teaching. The notion of the function is basic notion at the lower secondary level. We will present some examples from the national measurements in Slovakia (T9). The notion of the graph of the function and his graphical and visual representation is important at upper secondary level and university level. If the student draws the graph of the function, then the work with graphical representation of the function can help to make his knowledge about notions such continuity, derivative more deep. The development of information and communication technologies (ICT) gives the possibility to use different tools (for example educational software) as a supporting aspect for mathematics education with better understanding. We can find many suitable inspirative examples in the historical mathematical textbooks and materials. We would like to show some examples with the help of GeoGebra for motivational analysis teaching. This contribution is prepared with the help of the project VEGA 1/0079/19.

On quantitative assessment of In-situ and ex-situ composites structures with micro and nano-particles reinforcement

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The aim of the investigation is development of a quantitative methodology for In-situ and Ex-situ composites structures with micro and nano-particles reinforcement assessment. The primary method is based on the analytical representative volume element (ARVE) theory, which was developed to determine the effective properties of fiber composites. The analysis were carried out for two types of composites manufacturing methods (in-situ, ex-situ) and two types of particles (nano-TiC and micro-SiC). The developed methodology allows to compare composites with similar composition obtained through different technological processes. It also allows to assess the impact of process parameters on the composite structures. Moreover, the research methodology is applied to structure analysis of the in-situ composites.

R-linear problem and its application to random composites

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Let D_k be mutually disjoint simply connected domains bounded by smooth curves ∂D_k and D be the complement of all closures of D_k to the torus T^2. To find a function $\varphi(z)$ analytic in $D^+ = D_1 \cup \ldots \cup D_n$, $D^- = D$ and continuous in the closures of the considered domains on the torus T^2 with the following $\mathbf{R}$-linear conjugation condition

$$\varphi^+(t) = \varphi^-(t) - \rho(t)\overline{\varphi^-(t)}, \quad t \in \partial D^+,$$

where $\rho(t)$ is a constant on each component of ∂D^+. Application of the generalized method of Schwarz yields a constructive algorithm to solve the problem (see S. Gluzman, V. Mityushev, W. Nawalaniec, 2017). Relations to the Riemann-Hilbert problem are discussed. The obtained results are applied to description of composites, in particular, to the constructive RVE theory of random two-dimensional composites (see V. Mityushev, W. Nawalaniec, N. Rylko, 2018). We give precise

and computationally instant answers to such questions as isotropy of structure. Applications to metamaterials are discussed.

Classifying and analysis of random composites using structural sums feature vector

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We present the application of structural sums, mathematical objects originating from the computational materials science, in construction of a feature space vector of 2D random composites simulated by distributions of non-overlapping disks on the plane. Construction of the feature vector enables the immediate application of machine learning tools and data analysis techniques to random structures.

Trygonometric approach to the Schrödinger equation as a general case of known solutions

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General solutions of the Schrödinger equation were obtained by means of parametrization of the unit circle equation, related to the Riccati equation. Particular solutions, such as oscillator, Coulomb or Morse potentials, are given by appropriate choice of parameters. This concept is strictly connected with the theory of orthogonal polynomials and hypergeometric functions. As a result one can choose more objectively orthogonal basis in numerical computations, which leads to improvement of accuracy.

Analysis of the degree of image complexity used in eye tracking research on STEM education

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The article contains a description and a proposal of using mathematical methods to analyse the degree of image complexity for eye tracking research. Complex images are often used as stimuli for eye tracking experiments sometimes they do not have any distinctive nor important elements , e.g. works of art, illustrations, textbooks, websites. The article describes an attempt to apply mathematical methods, numerical algorithms, to analyse and determine the degree of image complexity. An attempt was made to search for correlations between the areas of images with the greatest changes of dynamics determined mainly by the analysis of the gradient of luminance and chrominance of image pixels and areas of interest experimentally determined using eye tracking methods. Research was carried out in the STEM education by an interdisciplinary group of cognitive didactic at the Faculty of Mathematics, Physics and Technical Sciences of the Pedagogical University of Cracow. The study was carried out using a remote stationary eye tracker.

Inverse problem for spherical particles and its applications to metal matrix composites reinforced by nano TiC particles

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Randomly distributed non-overlapping perfectly conducting n balls of radius R are embedded in a conducting matrix occupying a large ball of the normalized unit radius. The potential and the normal flux are given on the boundary of the unit ball. The locations of inclusions a_k are not known. The perturbation term of potential induced by inclusions is constructed up to $O(R^4)$ by a method of functional equations

$$U(\mathbf{x}) = \sum_{m=1}^{n} \left[|\mathbf{x}|^3 \frac{x_1 - |\mathbf{x}|^2 a_{m1}}{|\mathbf{x} - |\mathbf{x}|^2 \mathbf{a}_m|^3} - \frac{x_1 - a_{m1}}{|\mathbf{x} - \mathbf{a}_m|^3} \right]. \tag{1}$$

The perturbation term (1) includes the unknown centers of inclusions in symbolic form. The inverse problem is reduced to determination of the centers a_k by fitting of the given perturbation term on the unit sphere.

Three mental worlds of mathematics in pre-service mathematics teachers - an eye-tracking research

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Development of mathematical thinking can be considered in terms of "three mental worlds of mathematics", according to the theoretical framework formulated by D. Tall. These are: (1) a world of (conceptual) embodiment, (2) a world of (operational) symbolism and (3) a world of (axiomatic) formalism. These worlds should be developed by every secondary school and university student and a mathematics teacher should provoke their development during mathematical classes. The empirical research was designed in order to reveal some foundations of the process of problem solving. It will diagnose which of the three kinds of mathematical mental worlds was activated by the pre-service teachers in order to solve tasks. The presentation will be focused on understanding chosen notions (e.g. the notion of function and its representations) by pre-service mathematics teachers from this perspective. The presentation will discuss results of the research conducted with the use of eye-tracking technology at the Faculty of Mathematics and Physics and Technical Sciences of the Pedagogical University of Cracow.

Boundary integral equation method in the theory of binary mixtures of porous viscoelastic materials

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In this talk the linear theory for binary mixtures of porous viscoelastic materials is considered. The individual components of the mixture are a Kelvin-Voigt porous material and an isotropic elastic solid. The basic boundary value problems (BVPs) of steady vibrations and quasi-static are investigated. Namely, the fundamental solutions of the system of equations of steady vibrations and quasi-static are constructed explicitly and their basic properties are established. The uniqueness theorems for classical solutions of the internal and external basic BVPs of steady vibrations and quasi-static are proved. The surface and volume potentials are constructed and their basic properties are given. The determinants of symbolic matrices are calculated explicitly. The BVPs are reduced to the always solvable singular integral equations for which Fredholm's theorems are valid. Finally, the existence theorems for classical solutions of the internal and external BVPs of steady vibrations and quasi-static are proved by the boundary integral equation method and the theory of singular integral equations.

S.6 Fixed Point Theory, Ulam Stability, and Related Applications

Organisers
Erdal Karapinar, Janusz Brzdek

Scope of the session: It is an indisputable fact that both "Ulam Stability" and "Metric Fixed Point Theory" are among the most dynamic research fields of Nonlinear Analysis. Moreover, it has been clearly demonstrated that they are closely related to each other.
Ulam stability deals mainly with the following natural issue: when is it true that an approximate solution to an equation must be close to an exact solution of the equation. It is a quite new, but rapidly growing area of research with various possible applications. Motivated by the well-known problem of S. Ulam, concerning the approximate homomorphisms of metric groups, it has become nowadays an area of investigations of approximate solutions to a wide range of equations (e.g., difference, differential, integral, functional) and related fixed point results.
Due to its possible applications, Fixed Point Theory in the metric spaces plays a key role in Nonlinear Analysis. In the last fifty years, discussions on the existence and uniqueness of fixed points of single and multivalued operators in different kind of spaces (such as quasimetric spaces, pseudo-quasi-metric spaces, partial metric spaces, b-metric spaces and fuzzy metric spaces, among others) has attracted the attention of numerous researchers. The enormous potential of its applications to almost all quantitative sciences (such as Mathematics, Engineering, Chemistry, Biology, Economics, Computer Science, and others) justify the present great interest in this area. The purpose of this workshop is to bring together Mathematicians, but also other researchers that might be interested in this topic, to present, share and discuss their main advances (ideas, techniques, possible results, proofs, etc.) in this area.
Topics in this session include, but are not limited to: stability of difference, differential, functional, and integral equations, stability of inequalities and other mathematical objects, hyperstability and superstability, various (direct, fixed point, invariant mean, etc.) methods for proving Ulam's type stability results, generalized (in the sense of Aoki and Rassias, Bourgin and Gavruta) stability, stability on restricted domains and in various (metric, Banach, non-Archimedean, fuzzy, quasi-Banach, n-Banach, etc.) spaces, Ulam stability of operators, relations between Ulam's type stability and fixed point results, fixed point theory in various abstract spaces, existence and uniqueness of coupled/tripled/quadrupled fixed point, coincidence point theory, existence and uniqueness of common fixed points, well-posedness of fixed point results, advances on multivalued fixed point theorems, fixed point methods for the equilibrium problems and applications, iterative methods for the fixed points of the nonexpansive-type mappings, Picard operators on various abstract spaces, applications to various other areas.

—Abstracts—

A New Approach to Interval Matrices and Applications

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A convex combination of Intervals can be written as:

$$[a, b] = \{x_\alpha = (1 - \alpha)a + \alpha b : \alpha \in [0, 1]\}.$$

Consequently, we may adopt interval operations by applying the scalar operation point-wise to the corresponding interval points. With the usual restriction $0 \notin J$ if $\cdot = \div$. These operations are associative:

$$I + (J + K) = (I + J) + K,$$
$$I * (J * K) = (I * J) * K.$$

These two properties, which are missing in the usual interval operations, will enable the extension of the usual linear system concepts to the interval setting in a seamless manner. The arithmetic introduced here avoids such vague terms as "interval extension", "inclusion function", determinants which we encounter in the engineering literature that deal with interval linear systems. On the other hand, these definitions were motivated by our attempt to arrive at a definition of interval random variables and investigate the corresponding statistical properties. We feel that they are the natural ones to handle interval systems. We will enable the extension of many results from usual state space models to interval state space models. This feeling is reassured by the numerical results we obtained in a simulation examples.

Existence of Solutions to Boundary Value Problems for Intuitionistic Fuzzy Partial Hyperbolic Functional Differential Equations

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Intuitionistic partial functional differential equations are very rare, we combine two aspects, intuitionistic fuzzy mathematics and partial functional differential equations to get intuitionistic fuzzy partial hyperbolic functional differential equations, in this paper presents some new results on the existence and uniqueness of intuitionistic fuzzy solutions for some classes of intuitionistic fuzzy partial hyperbolic functional differential equations with integral boundary conditions using the Banach fixed point theorem. An illustrated example for our results is given with some numerical simulation for α-cuts of the intuitionistic fuzzy solutions.

A fixed point theorem in n-Banach spaces and Ulam stability

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In the talk, we give a fixed point theorem for operators acting on some classes of functions with values in n-Banach spaces. We also present its applications to the Ulam stability of eigenvectors and some functional and difference equations. The presented results were obtained jointly with Janusz Brzdek.

Impact of Perov type results on Ulam-Hyers stability

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Introducing a new kind of a contractive mapping which includes operator acting as a contractive constant gives us a possibility for a different approach on Ulam-Hyers stability of differential and operator equations. The application of several different fixed point theorems of Perov type will be presented along with some examples and comparison with previously obtained Ulam-Hyers stability results of these classes of equations.

Mittag-Leffler stability analysis for time-fractional partial differential equations

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In this work, we shall investigate a class of time-fractional Keller–Segel models with boundary Dirichlet conditions. We use Faedo–Galerkin method with some compactness arguments to show the existence results of weak solutions. Further, we establish the Mittag–Leffler stability of these weak solutions for the indicated models.

Application of F-contraction mappings to integral equations on time scales

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In this talk we present some existence and uniqueness of fixed points of certain $(\phi, F$-type contractions in the frame of metric like spaces. As an application of the theoretical results we consider the existence and uniqueness of solutions of nonlinear Fredholm integral equations of the second kind on time scales. We also discuss particular examples which demonstrate our theoretical findings and propose directions for further studies.

Fixed points of discontinuous mappings

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The goal of this study is to present and investigate some contractive type inequalities for discontinuous self-mappings . The main results cover and extend a few existing results in the corresponding literature. Furthermore, we give some illustrative examples to verify the effectiveness and strength of our derived results.

Local and global solution for effectively damped wave equations with non linear memory

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We prove the existence of local solutions for any data and global solution for small data for the Cauchy problem of effectively damped wave equation with non linear memory

$$u_{tt} - \Delta u + b(t)u_t = \int_0^t (t-s)^{-\gamma} |u(s,\cdot)|^p ds$$

$u(0,x)$ and $u_t(0,x)$ are given in suitably spaces.

Recent topics on metric fixed points

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The aim of this talk is present and discuss recent results and topics in the frame-work of metric fixed point theory. In addition, we talk possible applications of metric fixed point theory on distinct research areas.

Attractive Points by a Modern Iterative Process

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Takahashi and Takeuchi introduced the idea of attractive points. Since then some mathematicians have approximated (common) attractive points by classical iterative processes. Classical iterative processes include Picard, Mann, Ishikawa iterative processes and their multistep variants. A lot is still to be done in this direction. The author introduced an iterative process called Picard-Mann iterative process. This iterative process got a good attention of researchers and many generalizations came into show. The author in another paper introduced the ideas of common attractive points and further generalized mappings. He established existence of common attractive points and approximated them through a generalized version of the above-mentioned Picard-Mann iterative process. Here we continue this study by using a more modern iterative method to approximate attractive points of such mappings. A modern three-step iterative process may be defined as follows. Let $T : C \to C$ be a nonlinear mapping defined on a convex closed subset C of a Hilbert space H. Define the sequence $\{x_n\}$ iteratively as follows.

$$\begin{cases} x_{n+1} = Ty_n, \\ y_n = T((1-\delta_n)z_n + \delta_n Tz_n), \\ z_n = T((1-\mu_n)x_n + \mu_n Tx_n), \ n \in \mathbb{N}. \end{cases}$$

This iterative process converges faster than classical iterative processes. We prove some existence as well as convergence results for attractive points of T using the above iterative process. We also give a comparison of attractive and fixed points. This will improve and generalize several results including those of Khan.

Some related fixed point theorems for multivalued mappings on two metric spaces

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In this paper, we present some fixed point results for multivalued mappings on two complete metric spaces. First, we give a classical result which is an extension of the main result of Brain Fisher's related theorem given in 1981. After, considering the recent technique of Wardowski, we provide two related fixed point results for both compact and closed and bounded set-valued mappings via F-contraction type condition.

On best proximity point theory: From cyclic mappings to Tricyclic ones.

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The recently introduced Tricyclic mappings are self mappings defined on the union of three subsets A, B and C of a metric space and maps A into B, B into C and C into A. Taking inspiration from the recent work by the current authors. We shall discuss existence of best proximity points of both tricyclic contractions and tricyclic relatively nonexpansive mappings in different subclasses of metric spaces. First, we introduce the concept of (S) convex metric spaces, those are convex metric spaces whose convex structure verifies an additional condition, thereby, we acquire a best proximity point theorem for tricyclic contraction mappings. Afterwards , we study the structure of minimal sets of tricyclic relatively nonexpansive mappings in the setting of Kohlenbach hyperbolic spaces, this way we obtain an existence theorem for such mappings.

Ulam Stabilities for a Class of Higher Order Integro-Differential Equations

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We will identify sufficient conditions that allow us to guarantee different kinds of Ulam stabilities for a class of higher order integro-differential equations (within appropriate metric spaces). We will consider a so-called σ-semi-Hyers-Ulam stability, which is a kind of stability somehow between the Hyers-Ulam and the Hyers-Ulam-Rassias stabilities. Conditions are obtained in view to ensure Hyers-Ulam, σ-semi-Hyers-Ulam and Hyers-Ulam-Rassias stabilities for such class of integro-differential equations (which considers both cases of finite and infinite intervals of integration). Fixed point arguments and generalizations of the Bielecki metric have a central role in the proposed method. (Joint work with L. P. Castro.)

Some existence and convergence results for monotone mappings

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Fixed Point Theory is one of the most exciting branches of mathematics as it works as a bridge between pure and applied mathematics. Banach contraction principle is one the basic and most widely used result of mathematics. In 2004, Ran and Reurings generalized the Banach contraction principle to ordered metric spaces. Later in 2005, Nieto and Rodriguez used the same approach to further extend some more results of fixed point in partially ordered metric spaces and utilized them to study the existence of solution of differential equations. During this talk, we will discuss some recent existence and convergence results with respect to monotone nonexpansive mappings.
Keywords: Fixed Point, nonexpansive mappings, Banach contraction.
AMS-Classifications: 47H10, 54H25.

S.7 Function Spaces and Applications

Organisers
Alexandre Almeida, António Caetano,
Stefan Samko

Scope of the session: This session aims to cover recent progresses in the Theory of Function Spaces (including spaces of Lebesgue, Orlicz, Sobolev, Besov, Triebel-Lizorkin and Morrey-Campanato type) and their applications. Various generalizations are welcome, including spaces with variable exponents and others related to nonstandard growth conditions. Applications might range from properties of operators of harmonic analysis acting in such spaces to various applications to partial differential equations or integral equations. Applications in new contexts are also welcome.

—Abstracts—

Weighted Hardy type inequalities with a variable upper limit

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Let $0 < \alpha < 1$. The operator of the form

$$K_{\alpha,\varphi} f(x) = \int_a^{\varphi(x)} \frac{f(t)w(t)dt}{(W(x) - W(t))^{(1-\alpha)}}, \quad x > 0,$$

is considered, where the real weight functions $v(x)$ and $w(x)$ are locally integrable on $I := (a, b)$, $0 \le a < b \le \infty$ and $\frac{dW(x)}{dx} \equiv w(x)$.
In this paper we derive criteria for the operator $K_{\alpha,\varphi}$, $0 < \alpha < 1$, $0 < p; q < \infty$, $p > \frac{1}{\alpha}$ to be bounded and compact from the spaces $L_{p,w}$ to the spaces $L_{q,v}$.

Notes on bilinear multipliers on Orlicz spaces

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Let Φ_1, Φ_2, Φ_3 be Young functions and $L^{\Phi_1}(\mathbb{R})$, $L^{\Phi_2}(\mathbb{R})$, $L^{\Phi_3}(\mathbb{R})$ be corresponding Orlicz spaces. We say that a function $m(\xi, \eta)$ defined on $\mathbb{R} \times \mathbb{R}$ is a bilinear multiplier of type (Φ_1, Φ_2, Φ_3) if

$$B_m(f, g)(x) = \int_{\mathbb{R}} \int_{\mathbb{R}} \hat{f}(\xi)\hat{g}(\eta)m(\xi, \eta)e^{2\pi i(\xi+\eta)x}d\xi d\eta$$

is a bounded bilinear operator from $L^{\Phi_1}(\mathbb{R}) \times L^{\Phi_2}(\mathbb{R})$ to $L^{\Phi_3}(\mathbb{R})$. We denote by $BM_{(\Phi_1,\Phi_2,\Phi_3)}(\mathbb{R})$ the space of all bilinear multipliers of type (Φ_1, Φ_2, Φ_3) and investigate some properties of such a class and, under some conditions on the triple (Φ_1, Φ_2, Φ_3), give some examples of bilinear multipliers of type (Φ_1, Φ_2, Φ_3). We will focus on the case $m(\xi, \eta) = M(\xi - \eta)$ and get necessary conditions on (Φ_1, Φ_2, Φ_3) to get non-trivial multipliers in this class. In particular we recover some of the the known results for Lebesgue spaces.

On Besov Regularity of Solutions to Nonlinear Elliptic Partial Differential Equations

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We will be concerned with the regularity of solutions to nonlinear elliptic operator equatiions on domains of polygedral type. In particular, we study the smoothness in the specific scale $B^s_{\tau,\tau}, 1/\tau = s/d+1/p$ of Besov spaces The regularity in these spaces determines the approximation order that can be achieved by adaptive and other nonlinear approximation schemes. We show that for all cases under consideration the Besov regularity is high enough to justify the use of adaptive algorithms. The proofs are performed by using general embedding theorems between Kondratiev spaces and Besov spaces.

Characterizations of Besov spaces via K-functionals and ball averages

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Besov spaces occur naturally in many fields of analysis. In this talk, we discuss various characterizations of Besov spaces in terms of different K-functionals. For instance, we present descriptions via ball averages and minimization problems for bounded variation functions (Bianchini-type norms).
This is a joint work with S. Tikhonov (Barcelona).

Function spaces with general weights

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We introduce Besov and Triebel-Lizorkin spaces with general smoothness. These spaces unify and generalize the classical Besov and Triebel-Lizorkin spaces. We establish the φ-transform characterization of these spaces in the sense of Frazier and Jawerth and we prove their Sobolev embeddings. We study complex interpolation of these function spaces by using the Calderón product method and we identify their duals. We also we establish the smooth atomic, molecular and wavelet decomposition of these function spaces. To do these we need a generalization of some maximal inequality to the case of general weights.

Compact embeddings of Smoothness Morrey spaces on bounded domains

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We study the compact embedding between smoothness Morrey spaces on bounded domains and characterise its

entropy numbers. Here we discover a new phenomenon when the difference of smoothness parameters in the source and target spaces is rather small compared with the influence of the fine parameters in the Morrey setting. In view of some partial forerunners this was not to be expected till now. Our argument relies on wavelet decomposition techniques of the function spaces and a careful study of the related sequence space setting. This is joint work with Leszek Skrzypczak from Poznan.

Hölder continuity of quasiminimizers and ω-minimizers of functionals with generalized Orlicz growth

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I present recent results on local Hölder continuity of quasiminimizers of functionals with nonstandard (Musielak-Orlicz) growth. Compared with previous results, we cover more general minimizing functionals and need fewer assumptions. We prove Harnack's inequality and a Morrey type estimate for quasiminimizers. Combining this with Ekeland's variational principle, we obtain local Hölder continuity for ω-minimizers. This is joint work with Petteri Harjulehto and Mikyoung Lee.

Holomorphic Morrey, Orlicz, Grand Lebesgue and Hölder spaces

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We discuss some non standard spaces of functions holomorphic in domains on the complex plain: Orlicz, variable exponent Morrey, Grand/Small spaces and generalized Hölder spaces, constructed either with modulus of continuity or with variable exponent. We study boundedness of classical Bergman projection, growth of functions in these spaces near the boundary and some other questions.

Dual property of the Hardy-Littlewood maximal operator on reflexive variable Lebesgue spaces over spaces of homogeneous type

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We show that the Hardy-Littlewood maximal operator is bounded on a reflexive variable Lebesgue space $L^{p(\cdot)}$ over a space of homogeneous type (X, d, μ) if and only if it is bounded on its dual space $L^{q(\cdot)}$, where $1/p(x) + 1/q(x)$ for $x \in X$. This result extends the corresponding result of Lars Diening (2005) from the Euclidean setting of $\mathbb{R}^n$ to the setting of spaces of homogeneous type (X, d, μ).

Integral operators in mixed norm weighted function spaces and application

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The goal of our talk is to present the boundedness criteria for the fundamental integral operators of Harmonic Analysis in mixed weighted grand function spaces. We intend to give some applications to the approximation theory.
Acknowledgement. The work was supported by Shota Rustaveli National Science

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Maximal and Calderón-Zygmund operators in extrapolation Banach function lattices and applications

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We derived the boundedness of the Hardy-Littlewood maximal and Calderón-Zygmund operators in extrapolation Banach function lattices. As a consequence we have the boundedness of these operators in extrapolation spaces generated by Orlicz spaces over quasi-metric measure spaces with doubling measure. These results are applied to get the boundedness of the Calderón-Zygmund operator in grand Orlicz-Zygmund spaces with Muckenhoupt weights. The investigation was carried out jointly with V. Kokilashvili and M. Mastylo.
Acknowledgement. The work was supported by Shota Rustaveli National Science Foundation grant (No. FR-18-2499).

Embeddings of homogeneous Sobolev spaces on the entire space

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We completely characterize the validity of the inequality $\|u\|_{Y(\mathbb{R}^n)} \leq C\|\nabla^k u\|_{X(\mathbb{R}^n)}$, where X and Y are rearrangement-invariant spaces, by reducing it to a considerably simpler one-dimensional inequality. Furthermore, we fully describe the optimal rearrangement-invariant space on either side of the inequality when the space on the other side is fixed. We also solve the same problem within the environment in which the competing spaces are Orlicz spaces. A variety of examples involving customary function spaces suitable for applications is also provided.

Some embeddings of smoothness Morrey spaces in limiting cases

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The classical Morrey space $\mathcal{M}_{u,p}(\mathbb{R}^d)$, $0 < p \le u < \infty$, consists of all locally p-integrable functions f on $\mathbb{R}^d$ such that

$$\|f|\mathcal{M}_{u,p}(\mathbb{R}^d)\| = \sup_B |B|^{\frac{1}{u}-\frac{1}{p}} \left(\int_B |f(x)|^p\,dy\right)^{1/p}$$

is finite, where B runs over all balls in $\mathbb{R}^d$. These spaces are considered as an extension of the scale of L_p spaces, in view of $L_u(\mathbb{R}^d) = \mathcal{M}_{u,u}(\mathbb{R}^d) \hookrightarrow \mathcal{M}_{u,p}(\mathbb{R}^d)$ for any $p \le u$. Built upon these spaces Besov-Morrey spaces $\mathcal{N}^s_{u,p,q}(\mathbb{R}^d)$ and Triebel-Lizorkin-Morrey spaces $\mathcal{E}^s_{u,p,q}(\mathbb{R}^d)$ attracted some attention in last years, in particular in connection with Navier-Stokes equations. In this talk we focus on embeddings of these smoothness Morrey spaces in the borderline case of $s = n(1/u - p/u)_+$, and in the so-called critical case $s = d/u$. In the later case we obtain embeddings in Orlicz-Morrey spaces and in generalised Morrey spaces.
This is joint work with Dorothee Haroske (Jena) and Leszek Skrzypczak (Poznan).

Embeddings of Sobolev-type spaces into generalized Hölder spaces

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We present a sharp estimate for the$k$-modulus of smoothness, modelled upon a L^p-Lebesgue space, of a function f in $W^k L^{\frac{pn}{n+kp},p}(\Omega)$, where Ω is a domain with minimally smooth boundary and finite Lebesgue measure, $k, n \in \mathbb{N}$, $k < n$ and $\frac{n}{n-k} < p < +\infty$. This sharp estimate is used to establish necessary and sufficient conditions for continuous embeddings of Sobolev-type spaces into generalized Hölder spaces defined by means of the k-modulus of smoothness.
This is joint work with A. Gogatishvili and B. Opic.

Embedding properties for weighted Besov-Morrey and Triebel-Lizorkin-Morrey spaces

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We discuss Sobolev's embedding theorem on weighted Besov-Morrey and Triebel-Lizorkin-Morrey spaces with appropriate weight.

Boundedness of Riemann-Liouville operator from weighted Sobolev space to weighted Lebesgue space

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Work done in collaboration with Aigerim Kalybay. Let $I = (0,\infty)$, $1 < p, q < \infty$, $\frac{1}{p} + \frac{1}{p'} = 1$ and $\frac{1}{q} + \frac{1}{q'} = 1$. Let v, ρ and ω be functions nonnegative on I such that v^p, ρ^p, ω^q, $\rho^{-p'}$ and $\omega^{-q'}$ are locally summable on I.
Denote by $W^1_p(\rho, v) \equiv W^1_p(\rho, v, I)$ the space of all functions locally absolutely continuous on I having the finite norm

$$\|f\|_{W^1_p} = \|\rho f'\|_p + \|v f\|_p,$$

where $\|\cdot\|_p$ is the standard norm of the Lebesgue space $L_p(I)$.
Let $\overset{\circ}{AC}(I)$ be the set of all locally absolutely continuous functions with compact supports on I.
Denote by $\overset{\circ}{W}{}^1_p(\rho, v) \equiv \overset{\circ}{W}{}^1_p(\rho, v, I)$ the closure of the set $\overset{\circ}{AC}(I) \cap W^1_p(\rho, v)$ with respect to the norm of the space $W^1_p(\rho, v)$.
Let $L_{p,v} \equiv L_p(v, I)$ be the space of all measurable functions I with the finite norm $\|f\|_{p,v} \equiv \|v f\|_p$.
We consider the Riemann-Liouville fractional integration operator I_α:

$$I_\alpha f(x) = \int_0^x (x-s)^{\alpha-1} f(s)ds, \quad x \in I. \tag{0.1}$$

We establish a criterion for the boundedness of the Riemann-Liouville operator I_α from $\overset{\circ}{W}{}^1_p(\rho, v)$ to $L_{q,}(\omega, I)$, i.e., the fulfillment of the inequality:

$$\|\omega I_\alpha f\|_q \le C(\|\rho f'\|_p + \|v f\|_p), \quad f \in \overset{\circ}{W}{}^1_p(\rho, v).$$

Maximal regularity for local minimizers of non-autonomous functionals

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We establish local $C^{1,\alpha}$-regularity for some $\alpha \in (0,1)$ and C^α-regularity for any $\alpha \in (0,1)$ of local minimizers of the functional

$$v \mapsto \int_\Omega \varphi(x, |Dv|)\,dx,$$

where φ satisfies a (p,q)-growth condition. Establishing such a regularity theory with sharp, general conditions has been an open problem since the 1980s. In contrast to previous results, we formulate the continuity requirement on φ in terms of a single condition for the map $(x,t) \mapsto \phi(x,t)$, rather than separately in the x- and t-directions. Thus we can obtain regularity results for functionals without assuming that the gap between the upper and lower bounds is small, i.e. $\frac{q}{p}$ need not be close to 1. Moreover, for $\varphi(x,t)$ with particular structure, including p-, Orlicz-, $p(x)$- and double phase-growth, our single condition implies known, essentially optimal, regularity conditions. Hence we handle regularity theory for the above functional in a universal way.

Moser meets Gauss

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We present Moser-type estimates for Gaussian-Sobolev embeddings.

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n-best approximation in reproducing kernel Hilbert spaces

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The talk will introduce the n-best approximation model in reproducing kernel Hilbert spaces under a "doubling boundary vanishing condition". The background examples include Bergman and weighted Bergman and weighted Hardy spaces. In particular cases this method gives rise to rational approximation of the underlying space. The method can be extended to matrix-valued functions of certain hyper-complex variables.

Nonstandard bounded variation spaces

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We will discuss the notion of bounded variation spaces with variable exponent (in the Wiener and in the Riesz sense). In particular, we will show some embedding results, a Riesz representation lemma, and a characterization of global Lipschitz Nemytskii operators in the Riesz bounded variation spaces with variable exponent.

Boundedness and compactness of composition operators in holomorphic and harmonic spaces of Hölder type functions

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We study the boundedness and compactness of composition operators in the generalized Hölder type space of holomorphic functions in the unit disc with prescribed modulus of continuity and in the variable exponent generalized Hölder spaces of holomorphic functions in the unit disc.

Time-fractional telegraph equation

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In this work we obtain the first and second fundamental solutions (FS) of the multidimensional time-fractional equation with Laplace operator, where the two time-fractional derivatives of orders $\alpha \in]0,1]$ and $\beta \in]1,2]$ are in the Caputo sense. We obtain representations of the FS in terms of Hankel transform, double Mellin-Barnes integrals, and H-functions of two variables. As an application, the FS are used to solve Cauchy problems of Laplace type. Moreover, some considerations about its applications in future work and its L_p integrability will be presented.

Embeddings of weighted local generalized Morrey spaces into Lebesgue spaces on fractal sets

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We study embedding of weighted local generalized Morrey spaces defined on a quasi-metric measure set of general nature which may be unbounded, into Lebesgue spaces. In the general setting of quasi-metric measure spaces and arbitrary weights we give a sufficient condition for such an embedding. In the case of radial weights related to the center of local Morrey space, this general condition allows to obtain an effective sufficient condition in terms of (fractional in general) upper Ahlfors dimension of the set X: In this case we also obtain a necessary condition with the use of (fractional in general) lower Ahlfors dimension instead of the upper one, so that we have a criterion for the embedding when these dimensions coincide.

Lizorkin-Triebel-Morrey Spaces and Differences

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In my talk I plan to speak about the characterization of $F^{s,\tau}_{p,q}(R^d)$ by differences. At the beginning I will recall what is known about the characterization of the Lizorkin-Triebel spaces $F^s_{p,q}(R^d)$ itself and differences. In particular I will consider two types of characterizations by differences: (a) the so-called ball mean characterization and (b) the Strichartz characterization. Those characterizations are never valid for the maximal range of the parameters p, q, s and d. Usually s has to be sufficiently large depending on p, q and d. We will discuss some sufficient and some necessary conditions.

On Cantor's Λ functional and reconstruction of coefficients of multiple function series

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In our talk summable series with respect to the systems $\Phi = \left(\varphi_n(t)\right)_{n=0}^{\infty}$ of finite and measurable functions defined on $[0,1]$ by positive, regular, triangular Λ matrices are considered. It is introduced the notion of Cantor's Λ functional for Λ summable series. This notion generalizes, in particular, any trigonometric integral in the sense of the reconstruction of coefficients of the series. Reconstruction of coefficients of multiple function series by iterated using of Cantor's Λ functional is established. The work was fulfilled jointly with Tengiz Tetunashvili.
Acknowledgement. The work was supported by Shota Rustaveli National Science Foundation grant (No. DI 18-118).

Trigonometric approximation in weighted grand variable exponent Lebesgue spaces

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We intend to discuss a description of approximative subspaces of weighted grand variable exponent Lebesgue spaces. For the functions of spaces we plan to present the direct and inverse theorems of trigonometric approximation.
Acknowledgement. The work was supported by Shota Rustaveli National Science Foundation grant (No. DI 18-118).

Characterization of functions with zero traces from Sobolev spaces via the distance function from the boundary

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We characterize functions from Sobolev spaces with zero traces by conditions involving the distance from the boundary.

S.8 Function Spaces and their Applications to Nonlinear Evolutional Equations

Organisers
Mitsuru Sugimoto, Baoxiang Wang

Scope of the session: The session will discuss some recent progress in various Banach function spaces, especially arising from harmonic analysis, and their applications to nonlinear evolutional equations. This session is focusing on the following topics:
Time-frequency analysis
Modulation spaces, Besov spaces
Linear and nonlinear dispersive equations
Navier-Stokes, MHD equations

—Abstracts—

Global Existence and Uniqueness Analysis of Reaction-Cross-Diffusion Systems

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The global-in-time existence of weak and renormalized solutions to reaction-cross-diffusion systems for an arbitrary number of variables in bounded domains with no-flux boundary conditions are proved. The cross-diffusion part describes the segregation of population species and is a generalization of the Shigesada-Kawasaki-Teramoto model. The diffusion matrix is not diagonal and generally neither symmetric nor positive semi-definite, but the system possesses a formal gradient-flow or entropy structure. The reaction part is of Lotka-Volterra type for weak solutions or includes reversible reactions of mass-action kinetics and does not obey any growth condition for renormalized solutions. Furthermore, we prove the uniqueness of bounded weak solutions to a special class of cross-diffusion systems, and the weak-strong uniqueness of renormalized solutions to the general reaction-cross-diffusion cases.

Inhomogeneous Strichartz estimates in some critical cases

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Strong-type inhomogeneous Strichartz estimates are shown to be false for the wave equation outside the so-called acceptable region. On a critical line where the acceptability condition marginally fails, we prove substitute estimates with a weak-type norm in the temporal variable. We achieve this by establishing such weak-type inhomogeneous Strichartz estimates in an abstract setting. The application to the wave equation rests on a slightly stronger form of the standard dispersive estimate in terms of certain Besov spaces. This talk is based on a joint-work with Neal Bez and Sanghyuk Lee.

Wave packet transform and estimate of solutions to Schrödinger equations in modulation spaces

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In this talk, we give a representation of solutions to Schrödinger equations with potential via wave packet transform. By using this representation, we show the estimate of solutions to Schrödinger equations with time dependent sub-quadratic potential in modulation spaces and give a construction of solutions by time slicing method.

Boundedness of bilinear pseudo-differential operators with symbols in an $S_{0,0}$-type class

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In this talk, we extend the result proved by Miyachi–Tomita (2013), the $L^2 \times L^2 \to h^1$-boundedness of bilinear pseudo-differential operators with symbols belonging to the Hörmander class $BS^{-n/2}_{0,0}$, in two ways. First, we improve the target space h^1 to the amalgam space (L^2, ℓ^1), which is embedded into $h^1 \cap L^r$,

$1 < r \leq 2$, continuously. Secondly, we improve the the symbol class $BS^{-n/2}_{0,0}$ by replacing the weight function $(1 + |\xi_1|^2 + |\xi_2|^2)^{-n/2}$ by general functions. This talk is based on a joint research with Prof. Miyachi (Tokyo Woman's Christian University) and Prof. Tomita (Osaka University).

Ill-posedness of an NLS-type equation with derivative nonlinearity on the torus

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We consider a nonlinear Schrödinger equation with third-order dispersion and derivative nonlinearity, which is regarded as a mathematical model for the photonic crystal fiber oscillator. For the non-periodic problem, the associated Cauchy problem is known to be locally well-posed in Sobolev spaces. We show that in the periodic setting the derivative nonlinearity causes ill-posedness (more precisely, non-existence of local-in-time solutions) of the Cauchy problem in Sobolev spaces and even in Gevrey classes. This talk is based on a jount work with Yoshio Tsutsumi (Kyoto University)

Adomian Decomposition Method for Burgers' Equations

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In this paper we like to explore the full power of Adomian decomposition method (ADM) to solve nonlinear problems. ADM has the capability to obtain three different types of solutions, namely explicit solution, analytic solution and semi-analytic solution. When a closed form solution exists, ADM is possible to capture this explicit solution, while most of the numerical methods can only get approximation, not exact solution. On the other hand, ADM is itself a computational method for solution, while the method of characteristics needs assistance from other numerical methods to find characteristic curve and differential equation on it. We will demonstrate ADM to solve the prototype nonlinear PDEs, i.e. the inviscid and viscous Burgers' equations, and conclude the superior of ADM.

Operating functions on $A^q_s(\mathbf{T})$

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In this talk, we consider operating functions on $A^q_s(\mathbf{T})$. This is joint work with Enji Sato (Yamagata University).

Gradient estimates for heat equation in an exterior domain

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In this talk we consider gradient estimates for heat equation with Dirichlet boundary condition in an exterior domain. Our results describe the sharp time decay rates of the derivatives of solutions to the heat equation. As an application, we also consider the fractional Leibniz rule for the Dirichlet Laplacian in the exterior domain.
This talk is based on the joint work with Professor Vladimir Georgiev (University of Pisa).

Periodic ultra-distributions and periodic elements in modulation spaces

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In the present talk we characterize periodic elements in Gevrey classes, Gelfand-Shilov distribution spaces and modulation spaces, in terms of estimates of involved Fourier coefficients, and by estimates of their short-time Fourier transforms. We show that such spaces can be completely characterised in terms of formal Fourier series with suitable estimates on their coefficients. For periodic Gelfand-Shilov distributions such characterisations can be found in the literature in the case when the Gevrey parameter is strictly larger than 1. Our analysis is valid for all positive Gevrey parameters.
As a consequence, inverse problems for diffusion equations and similar equations on certain bounded domains can be handeled.
The proofs are based on new types of formulae of independent interestswhen evaluating the Fourier coefficients and which involve short-time Fourier transforms. The talk is based on a joint work with E. Nabizadeh.

Bilinear pseudo-differential operators with exotic symbols

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In this talk, we consider the boundedness of bilinear pseudo-differential operators with symbols in the bilinear Hörmander class $BS^m_{\rho,\rho}$, $0 \leq \rho < 1$. Our purpose is to determine the critical order $m = m(\rho, p, q)$ to assure the boundedness from products of Hardy spaces $H^p \times H^q$ to L^r, $1/p+1/q = 1/r$, in the full range $0 < p, q, r \leq \infty$.

A sparse bound for an integral operator with wave propagator

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The purpose of this talk is to give a sparse bound for

$$T_\ell f(x) := \int_1^2 \left| e^{it\sqrt{-\Delta}} \varphi_\ell(D) f(x) \right| dt$$

where $\ell \in \mathbb{N}$ and $\varphi_\ell(D) f := \mathcal{F}^{-1}[\varphi(\cdot/2^\ell)\hat{f}]$. This operator is related to the maximal Riesz means.

Global regularity for the 3D relativistic massive Vlasov-Maxwell system

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Given any smooth, suitably small initial data, which decays polynomially at infinity, we prove global regularity for the 3D relativistic massive Vlasov-Maxwell system. In particular, neither the initial distribution for particles nor the electromagnetic field is assumed to have a compact support. The compact support type assumption was assumed in all previous results.

Equivariant Schrödinger map from two dimensional hyperbolic spaces

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We consider the equivariant Schrödinger map from $\mathbb{H}^2$ to $\mathbb{S}^2$ which converges to the north pole of $\mathbb{S}^2$ at the origin and spatial infinity of the hyperbolic space. If the energy of the data is less than 4?, we show that the local existence of Schrödinger map. Furthermore, if the energy of the data sufficiently small, we prove the solutions are global in time. This is based on joint work with Jiaxi Huang.

S.9 Generalized Functions and Applications

Organisers
Michael Kunzinger,
Michael Oberguggenberger, Stevan Pilipović

Scope of the session: The session will be devoted to theory and application of generalized functions, which comprise, among others, distributions, ultradistributions, hyperfunctions and algebras of generalized functions. Applications include, but are not restricted to, linear and nonlinear partial differential equations, asymptotic analysis, geometry, mathematical physics, stochastic processes, and harmonic analysis, both in theoretical and numerical aspects. The session is open to contributions on any aspect of generalized functions and their applications.

—Abstracts—

H-distributions and variants

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H-distributions are microlocal defect functionals (like H-measures and semiclassical/Wigner measures, microlocal compactness forms), the objects which determine, in some sense, the lack of strong compactness for weakly convergent L^p sequences.
H-distributions are an extension of H-measures to the L^p–L^q setting, and so far they have been successfully applied in compactness by compensation theory with discontinuous coefficients and to velocity averaging. For their construction, the Plancherel theorem (which was sufficient for H-measures) had to be replaced by Hörmander-Mihlin's theorem for Fourier multipliers. In order to broaden their possible applicability one needs to develop some additional properties of H-distributions. In this, an appropriate variant of the Schwartz kernel theorem is crucial: it allows to identify a bilinear form on the space of test functions with a distribution of finite order in both variables; in fact, being a Radon measure in the physical x space, and the distribution of finite order in the dual ξ space.
In order to adjust H-distributions to some applications, we shall investigate different projections in the phase space, as well as different compactifications, comparing the constructions with that of microlocal compactness forms.

Characterization of wave front sets via multidimensional Stockwell transform

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The talk is dedicated to resolution of wave front sets using the multidimensional Stockwell transform defined with respect to the rotation group. The main results give the principles for directional smoothness, by providing criteria for regular directed points using the Stockwell transform. We work on the cases when the dimension is 1, 2, 4 and 8, and try to generalize the results on arbitrary dimension. The talk is based on a joint work with Katerina Saneva, Stevan Pilipović and Bojan Prangoski.

Applications of topological tensor products to the theory of distributions

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The theory of topological tensor products has been an important tool in the theory of distributions since the introduction of theory of vector-valued distributions by

L. Schwartz in 1957. We present a number of recent results in distribution theory which are shown using methods from the theory of topological tensor products. In particular, we will present some recent results on the convolvability and regularisation of (vector-valued) distributions.

On almost periodicity and almost automorphy

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This talk aims to give an overview on almost periodicity and asymptotic almost periodicity as well as almost automorphy and asymptotic almost automorphy in the framework of classical functions, distributions, ultradistributions and finally generalized functions.

Absence of remainders in the Wiener-Ikehara theorem

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The celebrated Wiener-Ikehara theorem is a cornerstone of Tauberian theory. It states:

Theorem. *Let S be a non-decreasing function and suppose that*

$$G(s) := \int_1^\infty S(x)x^{-s-1}\mathrm{d}x$$

converges for $\Re s > 1$ and that there exists a constant a such that $G(s) - a/(s-1)$ admits a continuous extension to $\Re s \geq 1$. Then

$$S(x) = ax + o(x).$$

If one wants to deduce a quantitive error term for $S(x) - ax$, one in general needs to ask greater regularity of the Mellin transform $G(s)$ of S. It is well known that one can get remainders if for example one assumes analytic continuation to $\Re s > \alpha$ for some α, $0 < \alpha < 1$, and certain bounds on $G(s) - a/(s-1)$ in the strip $\alpha < \Re s < 2$. In 2017, Müger raised the question if an explicit error term would follow from merely the analytic continuation (without assuming bounds) of $G(s) - a/(s-1)$. This was recently answered negatively by G. Debruyne and J. Vindas. Their proof uses methods from functional analysis such as the open mapping theorem, and is non-constructive.
In this talk, we will sketch how one can construct explicit counterexamples. This talk is based on collaborative work with Gregory Debruyne and Jasson Vindas.

Fujita's Blow-up property of Discrete Reaction-Diffusion Equations on Networks

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The purpose of this work is to study Fujita's blow-up property of the equation:

$$(E): \begin{cases} u_t = \Delta_\omega u + \psi(t)f(u), & \text{in } S \times (0,\infty), \\ u = 0, & \text{on } \partial S \times (0,\infty), \\ u(\cdot,0) = u_0, & \text{on } S, \end{cases}$$

In fact, we introduce a set $\Lambda(\psi)$(called a critical set) to prove the followings:

(i) if $f \in \Lambda(\psi)$, then the solutions to (E) blow up in finite time for every initial data.

(ii) if $f \notin \Lambda(\psi)$, then the solutions to (E) blow up in finite time for sufficiently large data.

(iii) if $f \notin \Lambda(\psi)$, then the solutions to (E) exist globally for small initial data.

Here,S is a finite network with boundary ∂S and weight ω. (This is a joint work with Dr. J.-H.Park and M.-J.Choi)

A Differential Calculus in the Framework Colombeau's Full Algebra

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Starting from the Colombeau's full generalized functions, the sharp topologies and the notion of generalized points, we introduce a new kind differential calculus. We study generalized pointvalues, Colombeau's differential algebra, holomorphic and analytic functions. We show that the Embedding Theorem and the Open Mapping Theorem hold in this framework. Moreover, we study some applications in differential equations.

This is joint work with Cortes, Wagner., da Silva, S. Horácio and Juriaans, O. Stanley.

Singularities in equations of fluid dynamics and applications to hurricanes.

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We prove new theorems concerning irregular solutions to PDE of fluid dynamics derived from Navier-Stokes equations in rotating coordinate systems. Such PDE are common in meterology, but we generalize our results to an arbitrary number of dimensions. Our solutions are composed of a regular part and singular part which is localized on the border of a compact domain in space. Our results comprise orthogonality conditions remeniscent of Rankine-Hugoniot type jump conditions. We apply our results to mathematical models of hurricanes to study in particular the dynamics near the eye of the hurricane. This also includes asymptotic expressions for the wind profile near the eye's wall.

Generalized functions in the optimal control of age-structured population models

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We use generalized functions in a qualitative optimization analysis of two age-structured population models and discuss the applied relevance of obtained theoretic results. Solutions from non-smooth functional spaces, including generalized functions, often appear in the optimal control of partial differential equations but are avoided in applications. A common consensus is that optimization problems with state constraints have weak solutions in wide classes with generalized functions. This talk demonstrates that generalized functions can arise in applied problems without any constraints just because of the nature of a process under study.
The first discussed problem maximizes the discounted concave profit from harvesting in the linear age-structured Lotka-McKendrick model. Such models often possess so-called bang-bang harvesting controls. We consider a relaxation of this optimal harvesting problem without upper bound on control variables and prove that optimal age-dependent harvesting control is expressed via the Dirac delta-function. The results establish new links between bang-bang and spiked harvesting strategies.
The second problem is the optimal distribution of investments into new and old equipment under evolving technology. A challenge is that solutions do not exist in normal functional classes, so, we have to employ generalized functions to construct solutions. The optimal capital and investment age-distributions involve the Dirac delta-function and its derivative. They are obtained as the limiting case of a related model with constant elasticity of substitution among equipment vintages of different ages. Numeric simulation illustrates analytic outcomes.

The eigenvalue problems for p-Schrödinger operators under the mixed boundary conditions on finite networks

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In this talk, we discuss the discrete eigenvalue problems for the p-Schrödinger equations under the mixed boundary conditions defined on networks as follows:

$$\begin{cases} -\Delta_{p,\omega}\phi + q|\phi|^{p-2}\phi = \lambda|\phi|^{p-2}\phi, & \text{in } S, \\ \mu\frac{\partial\phi}{\partial_p n} + \sigma|\phi|^{p-2}\phi = 0, & \text{on } \partial S, \end{cases}$$

where $p > 1$, q is a real-valued function on a network S, and μ, σ are nonnegative functions on the boundary ∂S of S, with $\mu(z) + \sigma(z) > 0$, $z \in \partial S$. Here, $\Delta_{p,\omega}$ and $\frac{\partial\phi}{\partial_p n}$ are discrete p-Laplace operator and discrete p-normal derivative, respectively. Next, we use the above result to provide the existence of a positive solution to the discrete Poisson equation

$$\begin{cases} -\Delta_{p,\omega}u + q|u|^{p-2}u = f, & \text{in } S, \\ \mu\frac{\partial u}{\partial_p n} + \sigma|u|^{p-2}u = 0, & \text{on } \partial S. \end{cases}$$

This is a joint work with Soon-Yeong Chung (sychung@sogang.ac.kr).

The convolution of ultradistributions in the context of their supports

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Various sufficient conditions for the existence and associativity of the convolution in the spaces of ultradistributions of Beurling and Roumieu types (as well as in the spaces of tempered ultradistributions of both types) are given in terms of their supports. The conditions are counterparts of the compatibility conditions of supports of distributions (and of the polynomial compatibility of supports of tempered distributions). Several new particular cases are discussed and new applications are shown.

Variational problems of Herglotz type with complex order fractional derivatives and less regular Lagrangian

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We derive optimality conditions for variational problems of Herglotz type whose Lagrangian depends on fractional derivatives of both real and complex order, and resolve the case of subdomain when the lower bounds of variational integral and fractional derivatives differ. Moreover, we consider a problem of the Herglotz type that corresponds to the case when the Lagrangian depends on the fractional derivative of the action and give an example of the problem that corresponds to the oscillator with a memory. Since our assumptions on the Lagrangian are weaker than in the classical theory, we analyze generalized Euler-Lagrange equations by the use of weak derivatives and the appropriate technics of distribution theory. Such an example is discussed in details. The talk is based on collaboration with T. M. Atanackovic and S. Pilipovic.

Parameter-dependent linear ODE and topology of domains

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The well-known solution theory for (systems of) linear ordinary differential equations undergoes significant changes when introducing an additional real parameter. Properties like the existence of fundamental sets of solutions or characterizations of such systems via nonvanishing Wronskians are sensitive to the topological properties of the underlying domain of the independent variable and the parameter. We give a complete characterization of the solvability of such parameter-dependent

systems in terms of topological properties of the domain, both classically and in the setting of Schwartz distributions. This is joint work with V.M. Boyko and R.O. Popovych.

Integro-differential equations for probabilistic characteristics of continuous and intermittent processes in spaces of distributions

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The talk covers a comparison of different approaches to obtaining integro-differential (pseudo-differential) equations for probabilistic characteristics of processes that occur under the influence of continuous and intermittent random perturbations.
• The approach based on existence of three limits for random processes on time interval Δt as $\Delta t \to 0$. They are limits related to the local first an second order moments of transition probability and a limit that separates continuous processes from discontinuous. Practically, existence of the limits is based on statistical data for probabilistic characteristics of random processes. We show the existence of these limits based on models leading to stochastic equations for the processes themselves.
• Semigroup approach to the Kolmogorov-Fokker-Planck and master equations. Construction of generators of pseudo-differential equations based on the Levy-Khintchin formula and the techniques of the generalized Fourier transform.
In the equations obtained formally there are generalized functions such as shifted δ-functions and even their products, as well as integral transforms of these functions. The special attention is paid to justification of these terms in spaces of generalized functions.

Existence results concerning the convolution of ultradistributions

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We consider and discuss various general sequential conditions (a) for integrability and (b) for convolvability of ultradistributions of Roumieu type in the space $\mathcal{D}'^{\{M_p\}}$, based on suitably chosen classes of approximate units. In both cases (a) and (b), we prove the equivalence of the considered conditions. Analogous results are obtained for tempered ultradistributions of Roumieu type in the space $\mathcal{S}'^{\{M_p\}}$. We also prove several existence theorems on the convolution in the spaces $\mathcal{D}'^{\{M_p\}}$ and $\mathcal{S}'^{\{M_p\}}$.

Supremum, infimum and hyperlimits of Colombeau generalized numbers

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It is well-known that the notion of limit in the sharp topology of sequences of Colombeau generalized numbers $\widetilde{\mathbb{R}}$ does not generalize classical results. In fact, the ring $\widetilde{\mathbb{R}}$ is non-Archimedean and this topology is generated by balls of infinitesimal radii. E.g. the sequence $\frac{1}{n} \not\to 0$ and a sequence $(x_n)_{n\in\mathbb{N}}$ converges *if* and only if $x_{n+1} - x_n \to 0$. This has several deep consequences, e.g. in the study of series, analytic generalized functions, or sigma-additivity and classical limit theorems in integration of generalized functions. The lacking of these results is also connected to the fact that $\widetilde{\mathbb{R}}$ is necessarily not a complete ordered set, e.g. the set of all the infinitesimals does not have neither supremum nor infimum.
We present a solution of these problems with the introduction of the *Robinson-Colombeau ring* ${}^{\rho}\widetilde{\mathbb{R}}$ and with the notion of *hypernatural number*

$$ {}^{\rho}\widetilde{\mathbb{N}} := \left\{ [n_\epsilon] \in {}^{\rho}\widetilde{\mathbb{R}} \mid n_\epsilon \in \mathbb{N}\ \forall \epsilon \right\}. $$

The ring ${}^{\rho}\widetilde{\mathbb{R}}$ is defined exactly as $\widetilde{\mathbb{R}}$, but where the generalized number $\rho = [\rho_\epsilon]$ plays the role of the classical $[\epsilon]$. The net $(\rho_\epsilon)_{\epsilon\in(0,1]} \downarrow 0$ is called a *gauge*. We define the notion of supremum and infimum and that of *Archimedean bound*, showing that the existence of the former is equivalent to that of the latter, which is, like in the classical case, an easier and convenient concept. In this way, we can generalize all the classical theorems for the *hyperlimit* of an *hypersequence* $(x_n)_n : {}^{\sigma}\widetilde{\mathbb{N}} \to {}^{\rho}\widetilde{\mathbb{R}}$, where $(\sigma_\epsilon)_\epsilon$ is another gauge: relations between hyperlimits and inequalities, Cauchy criterion, monotonic hypersequences, sub-hypersequences, limit superior and inferior, relations with sharp continuity of generalized functions, classical examples and counter-examples, etc. The fact that $\frac{1}{\log n} \to 0$ as $n \to +\infty$, $n \in {}^{\sigma}\widetilde{\mathbb{N}}\sigma$, for a certain gauge $\sigma > \rho$ but $\frac{1}{\log n} \not\to 0$ if $\sigma = \rho$ shows that the generalization using two gauges is necessary.
This is a joint work with D. Tiwari (University of Vienna), G. Apaaboah (Universit Grenoble Alpes) and P. Giordano (University of Vienna).

Generalized solutions of conservation law systems containing the delta distribution

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There is an interesting class of conservation law systems that posses unbounded solutions. We will describe some of them containing the Dirac delta function as a part of a solution. There is no unified way of treating these problems and we will present some of them, try to bring their good and bad sides and compare.

Asymptotic boundedness in ultradistribution spaces

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In this talk we study the structure imposed by asymptotic boundedness in ultradistribution spaces, which as usual is done by an analysis of its parametric behavior relative to some weight function. In the first part of the talk we will be considering convolution averages and provide a full characterization. Though such structural theorems have been found in the past, our approach is novel as we employ the short-time Fourier transform (STFT), which allows us to work under milder assumptions than those needed when working with the parametrix method. Because of this, the theory of the STFT had to be extended to the context of general ultradistributions.
In the second part of the talk, dilation is considered. By applying the previous results and certain techniques developed by Vindas and the speaker, a full characterization is at hand. This structure is then utilized to consider the so-called moment asymptotic expansion (MAE) in an ultradistributional context. A generalized function f is said to satisfy the MAE if there exists certain complex numbers $\{\mu_\alpha\}_{\alpha\in\mathbb{N}^d}$ such that the following asymptotic expansion holds

$$f(\lambda x) \sim \sum_{\alpha\in\mathbb{N}^d} \frac{(-1)^{|\alpha|}\mu_\alpha \delta^{(\alpha)}(x)}{\alpha!\lambda^{|\alpha|+d}}, \qquad \lambda\to+\infty.$$

In this talk we consider the MAE over several spaces of ultradifferentiable functions, providing a full characterization in the 1-dimensional case, whilst also considering a uniform analog.

The geometrization of the theory of full Colombeau algebras

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A natural field of application of Colombeau algebras of nonlinear generalized functions lies in general relativity, in particular in questions concerning metrics of low regularity where classical distribution theory fails. An early application of Clarke, Vickers and Wilson ('96) in this context involved the calculation of the curvature of conical metrics as an associated distribution. To free such calculations from the dependence on coordinate systems and special mollifiers, one naturally needs to develop full Colombeau algebras which are invariant under the action of diffeomorphisms.
I will give a broad overview of the steps involved in obtaining a diffeomorphism invariant theory of nonlinear generalized functions and how it is extended to the case of nonlinear generalized sections of vector bundles. Moreover, I will show how this structure accomodates virtually all previous variants of Colombeau algebras and can be used to calculate the curvature of conical metrics.

Propagation of singularities for generalized solutions to nonlinear wave equations

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The talk addresses propagation of singularities for nonlinear wave equations in the Colombeau algebra $\mathcal{G}$. While the subalgebra $\mathcal{G}^\infty$ is not suitable for nonlinear equations, the subalgebra $\mathcal{G}^0$ of Colombeau generalized functions all whose derivatives are of bounded type can be used. We first show that the Cauchy problem for nonlinear wave equations of the form $\Box u_\varepsilon = \varepsilon f(u_\varepsilon)$ can be globally solved in $\mathcal{G}^0$ up to three space dimensions. Here the nonlinear function f is assumed to be smooth and polynomially bounded; the nonlinearity is small in the sense that it is multiplied by the Colombeau parameter ε. Then we follow the classical road laid out by Michael Reed and Jeff Rauch in the 1970s showing that in one space dimension regularity propagates inside the light cone emanating from a singular initial point. Both results are proven by means of a fixed point theorem in the sharp topology.
The presentation is joint work with Hideo Deguchi.

Wave fronts-new results

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Wave fronts are defined for the distributions, for non-quasy-analytic Gevrey type ultradistributions of order $s > 1$, for quasy-analytic ultradistributions of order $s = 1$ (analytic wave front) and for their subspaces. Two new wave fronts: 1. for generalized functions between ultradistributions and distributions and 2. for quasy-analytic ultradistributions of order $s \in (1/2, 1)$, will be presented. Also the short time Fourier transformation is used for a new characterization of the classical wave front.
Key words and phrases: Wave front, ultradistributions
2010 Mathematics Subject Classification: 46F12, 46F05,

Ellipticity and the Fredholm property in the Weyl-Hörmander calculus

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In this talk we present results concentring the Fredholm properties and ellipticity of pseudodifferential operators with symbols in the Hörmander classes $S(M, g)$. The main result is that the Fredholm property of a ΨDO acting on Sobolev spaces in the Weyl-Hörmander calculus and the ellipticity are equivalent for geodesically temperate Hörmander metrics whose associated Planck functions vanish at infinity. Additionally, we prove that when the Hörmander metric is geodesically temperate, and consequently the calculus is spectrally invariant, the inverse $\lambda \mapsto b_\lambda \in S(1, g)$ of every $\mathcal{C}^N$, $0 \le N \le \infty$, map $\lambda \mapsto a_\lambda \in S(1, g)$ comprised of invertible elements on L^2 is again of class $\mathcal{C}^N$.

The talk is based on collaborative works with Stevan Pilipović

Aragona Algebras

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By introducing Distribution Theory, Schwartz was among the first to provide us with a well developed theory of generalized functions. This is a linear theory which does not admit multiplications among the new objects. Several attempts were made to find a setting which permits such multiplications while maintaining the essence of Schwartz' theory. This was achieved by Colombeau. Contributions to this new theory, made by many prominent researchers such as J. Aragona, H. Biagioni, M. Oberguggenberger, M. Kunzinger, E. Rosinger and Pilipovic among others, permitted that it became a well established theory. Algebraic and topological aspects of the theory was developed by many other researchers. The rings in which these new generalized functions take values are not fields and so the search for algebras of generalized functions having point values in a field became an interesting question. Examples in this direction were given by Vernaev and Todorov. These algebras are more in the direction of nonstandard analysis which is not such a surprise since Colombeau's algebras have a non-standard flavor. The starting point of this paper is the part of article from Khelif and Scarpalezos. There the similarity of some maximal ideals in $\mathcal{G}(\Omega)$ with maximal ideals in rings of continuous functions is noted. We develop all the machinery needed to carry out the details of the observation made in that paragraph. This is done in the setting of the full algebras of generalized functions. Along the way, we show that most models of algebras of generalized functions are quotients of Colombeau algebras. We name the arising algebras, Aragona algebras, in honor of Jorge Aragona. The results obtained are then used to study idempotents and ideals in the full algebra. While preparing the final draft of this paper we received the manuscript of Khelif and Scarpalezos with results in the same direction as ours for the simplified algebra.

This is joint work with Juriaans, O. S., Garcia, A. R. G., Silva, J. C. and is in honor of Professor Alfredo Jorge Aragona Vallejo.

Stochastic Fourier Integral Operators

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In this talk we will define Fourier integral operators (FIOs) with stochastic phase functions and stochastic amplitude functions. In general, one cannot easily prove measurability of stochastic FIOs. However, if one makes a small restriction the the phase function, one can show that the FIO depends continuously on the phase and amplitude function. This means that it is measurable. Finally, we will present some examples of stochastic Fourier integral operators coming from stochastic hyperbolic differential equations.

Some results on the zero set of Generalized Holomorphic functions

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A class of generalized functions was introduced by Colombeau. These are quotient of differentiable nets satisfying a growth condition induced by the parameter set. Here we are interested in those generalized functions which are nets of holomorphic functions. We take this as a definition but in fact it is a theorem.
It is well known that a holomorphic function of one variable can not have an accumulation point of zeros or poles in its domain. This is know as the Identity Theorem. This is no longer true in the context of generalized functions as the following simple example shows: let e^2=e and $f(z) = ez$. Then $z_n = (1-e)\alpha^n, \alpha = [\varepsilon \longrightarrow \varepsilon]$ is a sequence of zeros of f converging to 0.
Colombeau and Galé proved that if f is a generalized holomorphic function defined on Ω is zero on a subdomain of Ω than f is identically zero on Ω. These results were generalized by Khelif and Scarpalezos and they also give an example of a generalized function whose zero set is a dense G_δ set.
In this paper we consider sets one uniqueness E of the unit circle, i.e., the boundary ∂D, of the unit disc. By this we mean that if a function f which is continuous on the closure $\overline{D}$ of D and holomorphic in D is zero on E then f is identically zero. We prove the following

Theorem. *Let f be a generalized function defined on $\overline{D}$ and holomorphic in D and $E \subset \partial D$ a set of uniqueness. If $f_E = 0$ as a generalized function then $f \equiv 0$.*

This is joint work with Rodrigues, W. M., Juriaans, O. S. and Garcia, A.R.G and is in honor of Professor Alfredo Jorge Aragona Vallejo.

Hyperseries of Colombeau generalized numbers

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This talk is the natural succeeding step after the study of hypernatural numbers and hyperlimits (see talk of A. Mukhammadiev *Supremum, infimum and hyperlimits of Colombeau generalized numbers*).
Since the ring ${}^{\rho}\widetilde{\mathbb{R}}$ of Robinson-Colombeau is non-Archimedean, a classical series $\sum_{n=0}^{+\infty} a_n$ of generalized numbers $a_n \in {}^{\rho}\widetilde{\mathbb{R}}$ is convergent *if* and only if $a_n \to 0$ in the sharp topology. Therefore, this property does not permit us to generalize several classical results, mainly in the study of analytic generalized functions and in the study of sigma-additivity in integration of generalized functions. We first see that the reduction of the notion of convergent *hyperseries* to that of the hyperlimit of $N \in {}^{\sigma}\widetilde{\mathbb{N}} \mapsto \sum_{n=0}^{N} a_n \in {}^{\rho}\widetilde{\mathbb{R}}$ is possible only if $(a_n)_n \in \left(\mathbb{R}^{\mathbb{N}\times I}\right)_\rho / \sim_\rho =: {}^{\rho}\widetilde{\mathbb{R}}_{\mathrm{s}}$, where for $(a_{n\varepsilon})_{n,\varepsilon}$,

$(\bar{a}_{n\varepsilon})_{n,\varepsilon} \in \mathbb{R}^{\mathbb{N}\times I}$, we have $(a_{n\varepsilon})_{n,\varepsilon} \in \left(\mathbb{R}^{\mathbb{N}\times I}\right)_\rho$ if and only if

$$\exists Q \in \mathbb{N}\,\forall^0\varepsilon\,\forall n \in \mathbb{N}:\ |a_{n\varepsilon}| \leq \rho_\varepsilon^{-Q},$$

and $(a_{n\varepsilon})_{n,\varepsilon} \sim_\rho (\bar{a}_{n\varepsilon})_{n,\varepsilon}$ if and ony if

$$\forall q \in \mathbb{N}\,\forall^0\varepsilon\,\forall n \in \mathbb{N}:\ |a_{n\varepsilon} - \bar{a}_{n\varepsilon}| \leq \rho_\varepsilon^{q}.$$

We therefore need to ask for uniform moderateness and negligibility properties with respect to $n \in \mathbb{N}$. Using these notions, we recover classical examples such as ${}^\rho\sum_{n\in\sigma\widetilde{\mathbb{N}}} k^n = \frac{1}{1-k}$ for all $k \in {}^\rho\widetilde{\mathbb{R}}$, $0 < k < 1$; ${}^\rho\sum_{n\in\rho\widetilde{\mathbb{N}}} \frac{x^n}{n!} = e^x$ for all $x \in {}^\rho\widetilde{\mathbb{R}}$ finite, and convergence of ${}^\rho\sum_{n\in\rho\widetilde{\mathbb{N}}} \frac{1}{n^p}$ if $p > 1$. Also classical results can be proved, such as: algebraic operations on series and generalization of classical results for Cauchy product; $a_n \to 0$ (hyperlimit) if the series converges; relations with inequalities, absolute convergence, adding or removing an hyperfinite number of terms to/from an hyperseries and all the classical convergence tests. We finally briefly discuss about Cesáro summability and the formalization of classical divergent series using the hyperfinite sum $\sum_{n=0}^{N} a_n$, where $N \in {}^\rho\widetilde{\mathbb{N}}$ is an infinite hypernatural number.
This is a joint work with P. Giordano (Wolfgang Pauli Institute, Vienna).

Axiomatic Approach to Colombeau Theory

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We present a differential algebra of generalized functions over a field of generalized scalars by means of several axioms in terms of general algebra and topology. Our differential algebra is of *Colombeau type* in the sense that it contains a copy of the space of Schwartz distributions, and the set of regular distributions with $\mathcal{C}^\infty$-kernels forms a differential subalgebra. We discuss the uniqueness of the field of scalars as well as the consistency of our axioms. The talk is directed mostly for mathematicains who are not necessarily experts in Colombeau theory.
Key words: Algebraically closed field, real closed field, non-Archimedean ordered field, infinitesimals, saturated field, Cantor-complete field, valuation field, Schwartz distributions, Colombeau algebra.

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f-Frequently hypercyclic C_0-semigroups

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The main subject of the talk is f-frequently hypercyclic and q-frequently hypercyclic ($q \geq 1$) C_0-semigroups defined on complex sectors in the setting of separable Fréchet spaces. Results on generalized frequently hypercyclic translation semigroups and generalized frequently hypercyclic semigroups induced by semiflows on weighted spaces will be presented. Several illustrative related applications will be given.

Harmonic representations of generalized functions

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The purpose of this talk is to discuss an alternative way to construct linear generalized functions via the boundary values of harmonic functions. The procedure basically works for all classical generalized function sheaves between distributions and hyperfunctions, namely, for sheaves of ultradistributions and infrahyperfunctions. In particular, our ideas lead to a new elementary proof of Hörmander's support theorem for quasianalytic functionals. Several interesting new concepts will be introduced, such as that of "almost harmonic extensions" of functions. The talk is based on collaborative works with Andreas Debrouwere and Ricardo Estrada.

Some classes of stochastic evolution equations with Wick-type nonlinearities

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We study nonlinear stochastic evolution equations with Wick-power and Wick-polynomial type nonlinearities set in the framework of white noise analysis. In particular, these equations are of the form

$$u_t(t,\omega) = A\,u(t,\omega) + \sum_{k=0}^{n} a_k u^{\Diamond k}(t,\omega) + f(t,\omega),$$
$$u(0,\omega) = u^0(\omega), \quad \omega \in \Omega,$$

for $t \in (0,T]$, where $u(t,\omega)$ is an $X-$valued generalized stochastic process; X is a certain Banach algebra and A corresponds to a densely defined infinitesimal generator of a C_0-semigroup. These equations include the stochastic Fujita equation, the stochastic Fisher-KPP equation and the stochastic FitzHugh-Nagumo equation among many others. By implementing the theory of C_0-semigroups and evolution systems into the chaos expansion theory in infinite dimensional spaces, we prove existence and uniqueness of solutions for this class of SPDEs. Additionally, we treat the linear nonautonomous case and provide several applications featured as stochastic reaction-diffusion equations that arise in biology, medicine and physics.
The talk is based on collaborations with T. Levajković, S. Pilipović and D. Seleši.

S.10 Geometric & Regularity Properties of Solutions to Elliptic and Parabolic PDEs

Organisers
Pierre Bousquet, Lorenzo Brasco,
Rolando Magnanini

Scope of the session: In the study of partial differential equations, the quest for a precise description of the qualitative and quantitative features of their solutions have attracted much attention from the early beginnings. In recent years, once the classical question of well-posedness has been quite understood, research on topological and/or geometric properties of solutions to parabolic and elliptic PDEs have become more intense. The study of these aspects is tightly connected to that of Regularity Theory. Indeed, in many situations, one has to deal with solutions obtained by variational or viscosity methods - possibly in degenerate or singular regimes - which have to be intended in a suitable weak sense. Thus, a thorough analysis of the regularity of their solutions has to be undertaken in order to handle their geometric properties.
A list of topics currently and actively investigated in this field includes, to name a few: positivity; a priori estimates and sharp constants; critical points of solutions; symmetry and non-symmetry of ground states; rigidity results for overdetermined boundary value problems and entire solutions; stability of symmetric configurations; geometric properties of level sets of solutions; interplay between the curvature of the domain and the geometry of the relevant solutions.
The session is intended to put together young and expert researchers in these topics.

—Abstracts—

Minkowski inequality and nonlinear potential theory - 2

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In this talk, we first recall how some monotonicity formulas can be derived along the level set flow of the capacitary potential associated with a given bounded domain Ω. A careful analysis is required in order to preserve the monotonicity across the singular times, leading in turn to a new quantitative version of the Willmore inequality. Remarkably, such analysis can be carried out without any *a priori* knowledge of the size of the singular set. Hence, the same order of ideas applies to the p-capacitary potential of Ω, whose critical set, for $p \neq 2$, is not necessarily negligible. In this context, a generalized version of the Minkowski inequality is deduced. Joint works with M. Fogagnolo and L. Mazzieri.

Asymptotic regularity for a random walk over ellipsoids

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In 1979 Krylov and Safonov proved the Harnack inequality for solutions of the elliptic PDE in non-divergence form $\mathrm{Tr}\{A(x) \cdot D^2u(x)\} = 0$, where $A(\cdot)$ is a symmetric and uniformly elliptic matrix-valued function with measurable coefficients in $\Omega \subset \mathbb{R}^n$. Then the interior Hölder regularity of solutions followed as an immediate consequence. On the other hand, it turns out that viscosity solutions to this equation can be asymptotically characterized by means of a mean value property over a suitable family of ellipsoids. This establishes a connection between the PDE and a stochastic process describing a random walk in which the next step is chosen inside an (space-dependent) ellipsoid. Then the value functions of this process satisfy a *dynamic programming principle (DPP)*: a functional equation which reads as

$$u_\varepsilon(x) = \frac{1}{|E_\varepsilon(x)|} \int_{E_\varepsilon(x)} u_\varepsilon(y)\, dy,$$

where $E_\varepsilon(x)$ stands for an ellipsoid centered at x with size controlled by $\varepsilon > 0$ and shape determined by $A(x)$. We show that, provided with some uniform bound for the distortion of the ellipsoids, the solutions of the DPP are (locally) asymptotically Hölder continuous, that is

$$|u_\varepsilon(x) - u_\varepsilon(y)| \leq C\big(|x-y|^\alpha + \varepsilon^\alpha\big)$$

for all $x, y \in B_r \subset \Omega$, where $C > 0$ and $0 < \alpha < 1$ do not depend on ε.
Moreover, assuming some uniform bound for the maximum ratio between the eigenvalues of $A(x)$ and letting $\varepsilon \to 0$, this method provides an alternative proof for the interior Hölder regularity of certain viscosity solutions to $\mathrm{Tr}\{A \cdot D^2u\} = 0$. (Joint work with Mikko Parviainen, University of Jyväskylä).

Local regularity for viscosity solutions of quasilinear parabolic equations in nondivergence form

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In this talk we present some recent results on the regularity for a class of degenerate or singular parabolic equations in nondivergence from. We will see how to use the viscosity theory tools to prove Hölder estimates fo the gradient and provide qualitative properties of the viscosity solutions. Our equations are mainly modeled on the parabolic p-Laplace operator but similar equations can also be included.

Short-time asymptotics for solutions of the evolutionary game theoretic p-laplacian and Pucci operators

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Consider a domain $\Omega \subset \mathbb{R}^N$ with boundary $\Gamma \neq \varnothing$ and u the solution of $u_t - F\left(\nabla u, \nabla^2 u\right) = 0$ in $\Omega \times (0, \infty)$ such that $u = 1$ on Γ at all times and $u = 0$ in Ω at time $t = 0$. Here, we study the cases in which the differential operator F is either the game-theoretic p-laplacian or the Pucci operator.

We present two kinds of results concerning asymptotic formulas for small values of t. First, we associate appropriate rescalings of the values of $u(x,t)$ to the distance of x from the boundary Γ, generalizing what obtained by S. R. S. Varadhan when F is the Laplace operator. Secondly, we connect the asymptotic behavior of q-means on balls touching the boundary to a suitable function of principal curvatures of Γ at the touching point. These results generalize and extend formulas for the heat content, obtained by R. Magnanini and S. Sakaguchi in the linear case.
Asymptotic formulas also hold for $\varepsilon \to 0$, if one considers the solution u^ε of the elliptic equation $\varepsilon^2 F\left(\nabla u^\varepsilon, \nabla^2 u^\varepsilon\right) - u^\varepsilon = 0$ in Ω such that $u^\varepsilon = 1$ on Γ.
All results are based on joint work with R. Magnanini (Università di Firenze).

On the pairs of domains that solve a two-phase overdetermined problem of Serrin-type

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Let (D,Ω) be a pair of sufficiently smooth, bounded domains in the N-dimensional Euclidean space ($N \geq 2$). For two given positive constants $\sigma_c \neq \sigma_s$, we define $\sigma = \sigma(x)$ as the piece-wise constant function that takes the value σ_c in D and σ_s in $\Omega \setminus D$ respectively. We consider the following elliptic overdetermined problem of Serrin-type:

$$-\mathrm{div}(\sigma\nabla u) = 1 \ \ in\ \Omega,\ \ u = 0 \ \ on\ \partial\Omega,\ \ \sigma_s \partial_n u = c \ \ on\ \partial\Omega$$

Here $\partial_n u$ denotes the outward normal derivative of u, and c is a real constant. Notice that the overdetermined problem (1) does not admit a solution in general, but is solvable only for determinate pairs (D,Ω). We address the basic questions of (local) existence and uniqueness of such pairs and study their geometrical properties.

Some recent results in the study of fractional mean curvature flow

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We study a geometric flow driven by the fractional mean curvature. The notion of fractional mean curvature arises naturally when performing the first variation of the fractional perimeter functional. More precisely, we show the existence of surfaces which develope neckpinch singularities in any dimension $n \geq 2$. Interestingly, in dimension $n = 2$ our result gives a counterexample to Greyson Theorem for the classical mean curvature flow. We also present a very recent result, in the volume preserving case, establishing convergence to a sphere. The results have been obtained in collaboration with C. Sinestrari and E. Valdinoci.

Regularity and rigidity results for nonlocal minimal graphs

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Nonlocal minimal surfaces are hypersurfaces of Euclidean space that minimize the fractional perimeter, a geometric functional introduced in 2010 by Caffarelli, Roquejoffre, and Savin in connection with phase transition problems displaying long-range interactions. In this talk, I will introduce these objects, describe the most important progresses made so far in their analysis, and discuss the most challenging open questions. I will then focus on the particular case of nonlocal minimal graphs and present some recent results obtained on their regularity and classification in collaboration with X. Cabré (ICREA and UPC, Barcelona), A. Farina (Université de Picardie, Amiens), and L. Lombardini (UWA, Perth).

An optimization problem in thermal insulation

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Given a Lipschitz domain $D \subset \mathbb{R}^n$, and $\Omega = D + \delta B$, $\delta > 0$, let us consider the miminum problem

$$I_{\beta,\delta}(D) = \min\left\{ \int_{\Omega\setminus D} |\nabla\varphi|^2 dx + \beta \int_{\partial\Omega} \varphi^2 d\mathcal{H}^{n-1},\ \varphi \in H^1(\Omega),\ \varphi \geq 1 \right.$$

with $\beta > 0$. The minimum u of the functional $I_{\beta,\delta}(D)$ satisfies the following boundary value problem:

$$\begin{cases} \Delta u = 0 & \text{in } \Omega \setminus \bar{D}, \\ u = 1 & \text{su } \partial D, \\ \frac{\partial u}{\partial \nu} + \beta u = 0 & \text{su } \partial\Omega. \end{cases} \tag{0.2}$$

The equation (0.2) is related to a thermal insulating problem of a given domain D surrounded by an insulator $\Sigma = \Omega \setminus D$. The coefficient $\beta > 0$ depends on the physical properties of the insulator and the conditions outside of Ω; moreover, the insulator is distributed in such a way that its thickness is constant. The main aim is to determine the extremals of the functional $I_{\beta,\delta}(D)$ among the sets D in some suitable classes of domains. For example, in the planar case we consider the maximization problem

$$\sup_{D\subset\mathbb{R}^2:\ P(D)=k} I_{\beta,\delta}(D)$$

where δ, β and k are fixed positive numbers, and $P(D)$ denotes the perimeter of D. This is joint work with Carlo Nitsch (Università di Napoli Federico II) and Cristina Trombetti (Università di Napoli Federico II).

Strong unique continuation for higher order fractional Schrödinger equations

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In this talk I will show the strong unique continuation property for solutions of higher order fractional Schrödinger operators, including the case of variable coefficients and the presence of Hardy type gradient potentials. The proof relies on a generalised Caffarelli-Silvestre extension for the higher order fractional Laplacian and on Carleman estimates. I will also discuss applications of the unique continuation results in the context of nonlocal Calderón problems. This is a joint work with Angkana Rüland.

On a maximizing problem for the Moser-Trudinger type inequality with inhomogeneous constraints

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In this talk, we consider the existence or non-existence of a maximizer for the Moser-Trudinger type inequality in $\mathbb{R}^N$ of the form :

$$D_{N,\alpha}(a,b) := \sup_{\substack{u\in W^{1,N}(\mathbb{R}^N),\\ \|\nabla u\|_{L^N(\mathbb{R}^N)}^a+\|u\|_{L^N(\mathbb{R}^N)}^b=1}} \int_{\mathbb{R}^N} \Phi_N\left(\alpha|u|^{\frac{N}{N-1}}\right) dx,$$

where $N \geq 2$, $a, b > 0$, $\alpha \in (0, \alpha_N]$, $\Phi_N(t) := e^t - \sum_{j=0}^{N-2} \frac{t^j}{j!}$, $\alpha_N := N\omega_{N-1}^{1/(N-1)}$ and ω_{N-1} denotes the surface area of the unit ball in $\mathbb{R}^N$. We show the existence of the threshold $\alpha_* = \alpha_*(a,b,N) \in [0, \alpha_N]$ such that $D_{N,\alpha}(a,b)$ is not attained if $\alpha \in (0, \alpha_*)$, while is attained if $\alpha \in (\alpha_*, \alpha_N)$. We also provide the conditions on (a,b) such that the inequality $\alpha_* \in (0, \alpha_N)$ holds. In their proofs, we analyze the behavior of a maximizing sequence of $D_{N,\alpha}(a,b)$ to exclude the vanishing phenomenon which causes a non-compactness of the functional. This result is a joint work with Prof. Norihisa Ikoma in Keio University and Prof. Michinori Ishiwata in Osaka University, which is the sequel of the works in Ishiwata, Math.Ann.(2011), Ruf, J.Funct.Anal.(2005), Li-Ruf, Indiana Univ.Math.J.(2008) and do Ó-Sani-Tarsi, Commun.Contemp.Math.(2016).

Quasilinear parabolic equations: from Sturm attractors to Ginzburg-Landau patterns

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The study of the Ginzburg-Landau equations plays a key role in the dynamics of nonlinear fields. It serves as a normal form for PDEs near the Hopf instability, and is capable of displaying a vast array of patterns. We prove the existence of two types of patterns on the sphere: vortex and spirals solutions. On the other hand, the study of global attractors in the space of vortex solutions provides information about asymptotic dynamics of the system. Indeed, we show that such global attractors consist only of the vortex solutions (as equilibria) and their transition waves (as heteroclinics). Moreover, we construct explicitly the global attractors. In particular, we state necessary and sufficient conditions in order to determine which pair of vortex solutions admits a transition wave with pinned vortices.

Extremals for Morrey's inequality

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A celebrated result in the theory of Sobolev spaces is Morrey's inequality, which establishes in particular that for a bounded domain $\Omega \subset \mathbb{R}^n$ and $p > n$, there is $c > 0$ such that

$$c\|u\|_{L^\infty(\Omega)}^p \leq \int_\Omega |Du|^p dx, \quad u \in W_0^{1,p}(\Omega).$$

Interestingly enough the equality case of this inequality has not been thoroughly investigated (unless the underlying domain is $\mathbb{R}^n$ or a ball). I will discuss uniqueness properties of extremals of this inequality. These extremals are minimizers of the nonlinear Rayleigh quotient

$$\inf\left\{\frac{\int_\Omega |Du|^p dx}{\|u\|_{L^\infty(\Omega)}^p} : u \in W_0^{1,p}(\Omega) \setminus \{0\}\right\}.$$

In particular, I will present the result that in convex domains, extremals are determined up to a multiplicative factor. I will also explain why convexity is not necessary and why stareshapedness is not sufficient for this result to hold. The talk is based on results obtained with Ryan Hynd.

On Blaschke-Santaló diagrams involving the torsional rigidity and the first eigenvalue of the Dirichlet Laplacian

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In this talk we will present some recent results on some Blaschke-Santaló diagrams associated to the shape functionals λ_1 and T, the first Dirichlet eigenvalue and the torsional rigidity, respectively, under suitable geometric constraints. This is a joint work with D. Zucco (Politecnico di Torino, Italy).

The sharp quantitative isocapacitary inequality

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The well-known *isocapacitary inequality* states that balls minimize the capacity among all sets of the same given volume.
In the talk we prove a sharp quantitative form of this classical result. Namely, we show that the difference between the capacity of a set and that of a ball with the same volume bounds the square of the Fraenkel asymmetry of the set. This provides a positive answer to a conjecture of Hall, Hayman, and Weitsman.

Minkowski inequality and nonlinear potential theory - 1

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In this talk, we first recall how some monotonicity formulas can be derived along the level set flow of the capacitary potential associated with a given bounded domain Ω. A careful analysis is required in order to preserve the monotonicity across the singular times, leading in turn to a new quantitative version of the Willmore inequality. Remarkably, such analysis can be carried out without any *a priori* knowledge of the size of the singular set. Hence, the same order of ideas applies to the p-capacitary potential of Ω, whose critical set, for $p \neq 2$, is not necessarily negligible. In this context, a generalized version of the Minkowski inequality is deduced. Joint works with M. Fogagnolo and L. Mazzieri.

Hyperbolic solutions to Bernoulli's free boundary problem

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Bernoulli's free boundary problem is an overdetermined problem in which one seeks an annular domain such that the capacitary potential satisfies an extra boundary condition. This problem arises as the Euler-Lagrange equation for minimizing capacity among all subsets of equal volume in a prescribed container. There exist two different types of solutions: elliptic and hyperbolic solutions. Elliptic solutions are "stable" solutions and tractable by variational methods and maximum principles, while hyperbolic solutions are "unstable" solutions of which the qualitative behavior is less known. I will present a recent joint work with Antoine Henrot in which we show the existence and qualitative behavior of foliated hyperbolic solutions by a new flow approach.

Regularity for the normalized p-Poisson problem

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We discuss the local regularity of the normalized p-Poisson problem

$$-\Delta_p^N u = f \quad \text{in} \quad \Omega.$$

The normalized (or game-theoretic) p-Laplacian

$$\Delta_p^N u := |Du|^{2-p}\Delta_p u = \Delta u + (p-2)\Delta_\infty^N u$$

is in non-divergence form and arises for example from stochastic games. This is a joint work with Amal Attouchi and Mikko Parviainen.

Existence and regularity of Faber-Krahn minimizers in a Riemannian manifold

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We show a recent result about the existence and the regularity of sets whose minimize the first eigenvalue of the Laplace-Beltrami operator under volume constraint. Joint work with J. Lamboley.

On a thin coating problem and its related inverse problem

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In this talk, we consider an inverse problem related to a Robin eigenvalue problem and determine the Robin coefficients by the knowledge of the first eigenvalue and the measurement on some part of the boundary. This problem is closely related to the problem of thin insulator coating for a heat conductor. We prove the uniqueness of the inverse problem and establish the identification by using a Neumann tracking type functional. This is joint work with Matteo Santacesaria (University of Genoa).

A Quantitative Weinstock Inequality

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The aim of the talk is to present a result, obtained in a joint work with N. Gavitone, D. A. La Manna and G. Paoli, about a quantitative version of the Weinstock inequality in $\mathbb{R}^n$ with $n \geq 2$ for the first non trivial Steklov-Laplacian eigenvalue in the class of convex sets. The key role is played by a quantitative isoperimetric inequality which involves the boundary momentum, the volume and the perimeter of a convex set.

On the logarithmic epiperimetric inequality

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This talk will be dedicated to the logarithmic epiperimetric inequality, which is a new tool for the regularity of the free boundaries and the analysis of the singularities arising in the context of variational free boundary problems. In this talk we will consider the case of the classical obstacle problem and we will prove that the singular part of the free boundary is contained into the union of $C^{1,\log}$-regular manifolds.

S.11 Geometries Defined by Differential Forms

Organisers
Mahir Bilen Can, Sergey Grigorian, Sema Salur

Scope of the session: The subject of geometric structures on Riemannian manifolds is a very important, and intensely studied, subject in Differential Geometry. In particular, geometric structures such as contact geometry, symplectic geometry, calibrated geometry, and special holonomy geometries, are all defined by differential forms, and turn out to be very closely related to one another. Ever since the foundational work of Harvey and Lawson in 1982, the relationship between calibrated geometry and special holonomy has been of fundamental importance in Differential Geometry, as any $G-$invariant $p-$form on $\mathbb{R}^n$ induces a calibration on a connected Riemannian manifold (M^n, g) with holonomy $G \subset O(n)$. Moreover, recent directions in research show that there are manifolds that lie at the intersection of contact geometry and special holonomy, in particular G_2 and $Spin(7)$ holonomy manifolds, and that many ideas from both contact geometry and symplectic geometry have analogues to manifolds with special holonomy. Many of these objects have complexifications which are amenable to algebraic geometric techniques. In turn, real algebraic analogues provide a new frontier for research.
The cross-pollination of ideas from different areas of geometry takes roots in papers such as those of Fernandez and Gray from the 1960's wherein manifolds with special holonomy are treated as analogues of Kähler manifolds; moreover, because of the introduction in recent years of such powerful tools in symplectic and contact geometries such as Floer homology theories and its relatives, it is more important than ever to understand the connections between these fields as this will yield deeper results about manifolds with special holonomy and calibrated geometries than have been possible up to this point.
Our goal is to bring together a group of both junior and senior mathematicians who are experts in these different geometric structures in order to create a venue where they can interact and exchange ideas and developments, and to further the understanding of the relationships between these geometries, as well as the implications in gauge theory, (real) algebraic geometry, geometric PDE's, and mathematical physics.

—Abstracts—

The Asymptotic Nilpotent Variety of G2

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The asymptotic semigroup of a complex semisimple Lie group H is the algebraic variety that is obtained from H by taking its horospherical contraction. This semigroup, which is originally introduced by Vinberg, is closely related to the canonical compactification of the adjoint form of H. In this talk, by building on the works of Putcha, Renner, Rittatore, and Therkelsen, we will introduce and study the nilpotent variety associated with the asymptotic semigroup of H. We will demonstrate our results on the exceptional Lie group H:=G2.

A compact G_2-calibrated manifold with first Betti number $b_1 = 1$.

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We construct a compact formal 7-manifold with a closed G_2-structure and with first Betti number $b_1 = 1$, which does not admit any torsion-free G_2-structure, that is, it does not admit any G_2-structure such that the holonomy group of the associated metric is a subgroup of G_2. We also construct associative calibrated (hence volume-minimizing) 3-tori with respect to this closed G_2-structure and, for each of those 3-tori, we show a 3-dimensional family of non-trivial associative deformations. We also construct a fibration of our 7-manifold over $(S^2 \times S^1)$ with generic fibre a (non-calibrated) coassociative 4-torus and some singular fibers. Joint work with Marisa Fernández, Anna Fino and Vicente Muñoz.

Ancient solutions in Lagrangian mean curvature flow

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Motivated by mirror symmetry, the Thomas-Yau conjecture states that the convergence of almost calibrated Lagrangian mean curvature flow (LMCF) to a special Lagrangian is governed by a "stability condition", as one has for Hermitian-Yang-Mills flow. Singularities of almost calibrated LMCF are necessarily Type II, and so modelled by ancient solutions to the flow. I will describe classification results for almost calibrated ancient solutions to LMCF, which give us new understanding of singularity formation for the flow. This is joint work with Ben Lambert and Felix Schulze.

Generalizing holomorphic bundles to almost complex 6-manifolds

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(Joint work with Gavin Ball) The notion of a holomorphic bundle and Hermitian-Yang-Mills connections is one that proved to be very fruitful in complex geometry. There are some natural generalizations of these notions in almost complex geometry such as those of pseudo-holomorphic and pseudo-Hermitian-Yang-Mills connections. In this talk, I will focus on a system of partial differential equations, the DT-instanton equations, whose solutions give a further generalization of the notion of a Hermitian-Yang-Mills connection in the setting of real 6 dimensional almost Hermitian manifolds.

Extremal length in higher dimensions and complex systolic inequalities

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Extremal length is a classical tool in 1-dimensional complex analysis for building conformal invariants. We propose a higher-dimensional generalization for complex manifolds defined in terms of complex volume forms, and provide some ideas on how to estimate and calculate it. We also show how to formulate natural geometric inequalities in this context in terms of a complex analogue of the classical Riemannian notion of systole.

Generalized geometry in physics and mechanics

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In this talk I will present various instances of the generalized geometry, appearing naturally in some fields of contemporary physics and mechanics. From the mathematical perspective, I will mostly discuss Dirac structures - those generalize simultaneously Poisson, symplectic geometry and more. I will also recall the description of them in terms of graded geometry and differential graded manifolds. From the "applied" point of view, on the one hand, it turns out to be a convenient language for theoretical physics [1-3]. It permits to study some sigma models, define the gauging procedure in terms of equivariant cohomology, and relate the algebra of symmetries of the respective functionals to multi-symplectic geometry. On the other hand, for mechanics one can also spell-out the Dirac structures in the context of dissipative and coupled systems [4]. Time permitting, I will also explain that those can be important for structure preserving integrators.
References: [1] V.Salnikov, T.Strobl, Dirac Sigma Models from Gauging, Journal of High Energy Physics, 11/2013 ; 2013(11) [2] A.Kotov, V.Salnikov, T.Strobl, 2d Gauge Theories and Generalized Geometry, Journal of High Energy Physics, 2014:21, 2014 [3] V.Salnikov, Graded geometry in gauge theories and beyond, Journal of Geometry and Physics, Volume 87, 2015 [4] V.Salnikov, A.Hamdouni, From modelling of systems with constraints to generalized geometry and back to numerics, Z Angew Math Mech. 2019

Almost formality of closed G_2- and Spin(7) manifolds

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We introduce the notion of Poincaré DGCAs of Hodge type, which is a subclass of Poincaré DGCAs encompassing the de Rham algebras of closed orientable manifolds. We show that a $(r-1)$ connected Poincaré DGCA of Hodge type $\mathcal{A}^*$ of dimension $n \leq 5r-3$ is A_∞-quasi-isomorphic to an A_3-algebra and prove that the only obstruction to the formality of $\mathcal{A}^*$ is a distinguished Harrison cohomology class $[\mu_3] \in \mathsf{Harr}^{3,-1}(H^*(\mathcal{A}^*), H^*(\mathcal{A}^*))$. Moreover, the cohomology class $[\mu_3]$ and the DGCA isomorphism class of $H^*(\mathcal{A}^*)$ determine the A_∞-quasi-isomorphism class of $\mathcal{A}^*$. This can be seen as a Harrison cohomology version of the Crowley-Nordström results on rational homotopy type of $(r-1)$-connected $(r>1)$ closed manifolds of dimension up to $5r-3$. We also derive the almost formality of closed G_2-manifolds, which have been discovered recently by Chan-Karigiannis-Tsang from our results and the Cheeger-Gromoll splitting theorem. This is a joint work with Domenico Fiorenza, Kotaro Kawai, and Lorenz Schwachhofer in arXiv:1902.08406.

On a sub-Riemannian space associated to a Lorentzian manifold

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I will present a certain construction of a sub-Riemannian space naturally associated to a Lorentzian manifold. Some additional structures and relations between geometric properties of the corresponding spaces will be explored.

Geometric flows of closed forms

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The talk focuses on geometric flows of closed forms on smooth manifolds. This class of flows includes some interesting flows in literature, as the Laplacian flow in G2-geometry. In the talk it will be described a general criterium to establish if a flow of closed forms is well-posed and it will be discussed the stability around static solutions.
The talk is mainly based on the following papers written in collaboration with Lucio Bedulli
[1] L. Bedulli, L. Vezzoni, A parabolic flow of balanced metrics, J. Reine Angew. Math. 723 (2017), 79-99.
[2] L. Bedulli, L. Vezzoni, Stability of geometric flows of closed forms, arXiv:1811.09416.

Classifying Spaces of Diffeological Groups

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Fix an irrational number A, and consider the action of the group of pairs of integers on the real line defined as follows: the pair (m,n) sends a point x to $x+m+nA$. Since the orbits of this action are dense, the quotient topology on the orbit space is trivial, and continuous real-valued functions are constant. Can we give the space any type of useful "smooth" group structure?
The answer is "yes": its natural diffeological group

structure. It turns out this group is not just some pathological example, but has many interesting associated structures, and is of interest to many areas of mathematics. In particular, it shows up in geometric quantisation and the integration of certain Lie algebroids as the structure group of certain principal bundles, the main topic of this talk.
We will perform Milnor's construction in the realm of diffeology to obtain a diffeological classifying space for a diffeological group G, such as the irrational torus. After mentioning a few hoped-for properties, we then construct a connection 1-form on the G-bundle $EG \to BG$, which will naturally pull back to a connection 1-form on sufficiently nice principal G-bundles. We then look at what this can tell us about irrational torus bundles, their connection 1-forms, and corresponding curvature forms. [Joint work with Jean-Pierre Magnot.]

Contact Structures on G_2 Manifolds

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In 2011, Arikan, Cho, and Salur introduced the concept of $A-$ and $B-$compatible contact structures on G_2 manifolds. I will explain the relationship between symplectic, (almost) contact, and G_2 structures on manifolds with G_2 structures from this new perspective. I will give some examples of compatible structures and explain some new results about these relationships.

S.12 Harmonic Analysis and Partial Differential Equations

Organisers
Vladimir Georgiev, Tohru Ozawa,
Michael Ruzhansky, Jens Wirth

Scope of the session: The session will focus on the interplay between harmonic analysis and the theory of partial differential equations, highlight recent advances and bring together experts in both fields. Topics covered in the session will be sharp constants in variational inequalities, relations to solvability/stability of (nonlinear) partial differential equations, dispersive properties and scattering of equations mathematical physics, symmetry based concepts, aspects of harmonic analysis on Lie groups and related operator quantisations.

—Abstracts—

On Kohn-Nirenberg symbols of operators on the Heisenberg group

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We examine Kohn-Nirenberg symbols in relation to their operators on the Heisenberg group. Our goal is to gain criterias for an operator, such that its symbol is uniformly bounded. Estimates of this form are of importance to Beals-Cordes type characterizations of certain algebras of pseudodifferential operators, defined by Michael Ruzhansky and Vernonique Fischer. The used methods involve a Schwartz-kernel representation of both the Kohn-Nirenberg quantization and the Fourier transform on the Heisenberg group with respect to alternative spaces of test functions. These test functions are either defined to be usual Schwartz functions, which are orthogonal to all polynomials in the center coordinates of the Heisenberg group, or defined to be operator valued Schwartz functions vanishing rapidly in zero.
Also we find, analogously to the compact group case, a representation of the Kohn-Nirenberg symbols with the help of Fourier multipliers given by vector valued, slowly increasing functions.

Focusing energy-critical inhomogeneous NLS

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In this talk we consider the global well-poseness of focusing energy-critical inhomogeneous NLS. Based on the Kenig-Merle method we find a sufficient condition on the initial data associated with inhomogeneous term. This gives us GWP and scattering in $\dot{H}^1$. For this purpose we develope long time perturbation, profile decomposition, and rigidity theorem.

Critical well-posedness for the modified Korteweg-de Vries equation and self-similar dynamics

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We consider the modified Korteweg-de Vries equation over the real line

$$u_t + u_{xxx} = \pm(u^3)_x.$$

This equation arises, for example, in the theory of water waves and vortex filaments in fluid dynamics. A particular class of solutions to (mKdV) are those which do not change under scaling transformations, the so-called *self-similar* solutions. Self-similar solutions blow-up when $t \to 0$ and determine the asymptotic behaviour of the evolution problem at $t = +\infty$.
The known local well-posedness results for the (mKdV) fail when one considers critical spaces, where the norm is scaling-invariant. This also means that self-similar solutions lie outside of the scope of these results. Consequently, the dynamics of (mKdV) around self-similar solutions are currently unknown.
In this talk, we will show existence and uniqueness of solutions to the (mKdV) lying on a critical space which includes both regular and self-similar solutions. Afterwards, we present several results regarding global existence, asymptotic behaviour at $t = +\infty$ and blow-up phenomena at $t = 0$.

Old and New Trends in Hausdorff Operator

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Liflyand and Móricz studied the Hausdorff operator on the real Hardy space and proved the boundedness property and the commuting relations of this operator and Hilbert transformations. The main objective of this paper is extending these results to the two contexts of Dunkl theory and the Jacobi hypergroup. (This talk is based on the joint work with T. Kawazoe and F. Saadi)

Global almost radial solutions to supercritical dispersive equations outside the unit ball

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I consider a defocusing semilinear wave equation with power nonlinearity, on the exterior of the unit ball, with Dirichlet boundary conditions, in dimension $n \geq 3$. For any radial initial data, the solution is global and decays as t^{-1}. I prove that the radial solutions are stable for small perturbations of the initial data in a weighted Sobolev norm of order O(n) of Christodoulou type, and for sufficiently high powers $p >$O(n). This produces a fanily of global large solutions to the supercritical wave equation on the exterior of the ball, with arbitrarily high power nonlinearity. The almost radial solutions thus constructed are unique among energy class solutions which satisfy an energy inequality, by a weak-strong stability result of Struwe type. Similar results hold for the defocusing supercritical nonlinear Schrodinger equation.

Remark on blow-up for the half Ginzburg-Landau-Kuramoto equation with rough coefficients

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In this talk, we study the blow-up for solutions to the half Ginzburg-Landau-Kuramoto equation with an ode argument The key tools in the proof are appropriate commutator estimates and the essential self-adjointness of the symmetric uniformly elliptic operator with rough metric.

Uniqueness and nondegeneracy of ground states for nonlinear Schrödinger equations with attractive inverse-power potential

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We consider the uniqueness and nondegeneracy of ground states for stationary nonlinear Schrödinger equations with a focusing power-type nonlinearity and an attractive inverse-power potential. In this talk, we prove that all ground states are positive up to phase rotation, radial, and decreasing. Moreover, by extending the results of Shioji and Watanabe (2016), we prove the uniqueness and nondegeneracy of the positive radial solutions.

Uniqueness of standing-waves solutions to the nonlinear Schroedinger equations for combined power-type non-linearities

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We illustrate recent results of orbital stability for nonlinear dispersive equations. One of the problems we will be focusing on is the stability of standing-wave solutions to the non-linear Schrödinger equation

$$i\partial_t\varphi(t,x) + \Delta_x\varphi(t,x) - G(\varphi(t,x)) = 0.$$

The stability can be deduced from properties of the set of minima of the functional

$$E(u) = \frac{1}{2}\int_{\mathbb{R}^n} |\nabla u(x)|^2 dx + \int_{\mathbb{R}^n} G(u(x))dx$$

on the constraint $S = \{u \in H^1_{r,+}(\mathbb{R}^n) \mid \|u\|^2_{L^2} = 1\}$. The functions $H^1_{r,+}(\mathbb{R}^n)$ are radially symmetric (with respect to the origin), H^1 and positive. Orbital stability holds when there are finitely many minima. It is known from (M. K. Kwong, ARMA 1989) and (H. Berestycki and P. L. Lions, ARMA, 1983), that if $G(s) = -a|s|^p$, there is exactly one minimum. In this talk we present an extension of this uniqueness result to *combined power-type* non-linearities, $G(s) = -a|s|^p + b|s|^q$ and spatial dimension $n = 1$. We also provide a condition on G which guarantees that minima of the functional E on S are non-degenerate. As a consequence, there are only finitely many of them.

Scattering solutions for the focusing nonlinear Schrödinger equation with a potential

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This talk is based on a joint work with Masahiro Ikeda from RIKEN AIP, Japan. We consider the nonlinear Schrödinger equation with a linear potential term:

$$(\mathrm{NLS}_V)\begin{cases} i\partial_t u + \Delta u - Vu = -|u|^{p-1}u, & (t,x) \in \mathbb{R}\times\mathbb{R}^3, \\ u(0,x) = u_0(x) \in H^1(\mathbb{R}^3), \end{cases}$$

where $V = V(x)$ is a real-valued potential. We determine the conditions for the time behaviors of a solution to (NLS_V), scattering, blowing-up, and growing-up. The conditions are expressed in terms of some quantities which consist of mass and energy. Sharp criteria

are given by these quantities of the ground state of the standard NLS, without the potential. If we state more precisely, we give a sufficient condition of the potential and initial data for scattering solutions, and this is a main topic in this talk. Also, we introduce sufficient conditions of the potential and initial data such that blowing-up or growing-up holds at least. Moreover if we add some conditions, we can prove that the solutions must blow-up.

Probabilistic well-posedness of the mass-critical NLS with radial data below $\mathbb{R}^d$

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In this talk, we consider the Cauchy problem of the mass-critical nonlinear Schrödinger equation (NLS) with radial data below $L^2(\mathbb{R}^d)$. We prove almost sure local well-posedness along with small data global existence and scattering. Furthermore, we also derive conditional almost sure global well-posedness of the defocusing NLS under the assumption of a probabilistic *a priori* energy bound. The main ingredient is to establish the probabilistic radial Strichartz estimates.

Test function method for blow-up phenomena of semilinear wave equations and their weakly coupled systems

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In this talk we consider the wave equations with power type nonlinearities including time-derivatives of unknown functions and their weakly coupled systems. We propose a framework of test function method and give a simple proof of the derivation of sharp upper bound of lifespan of solutions to nonlinear wave equations and their systems. We point out that for respective critical case, we use a family of self-similar solution to the standard wave equation including Gauss's hypergeometric functions which are originally introduced by Zhou (1992). However, our framework is much simpler than that. As a consequence, we found new (p, q)-curve for the system $\partial_t^2 u - \Delta u = |v|^q, \partial_t^2 v - \Delta v = |\partial_t u|^p$ and lifespan estimate for small solutions for new region.

The Strichartz estimates for the damped wave equation and its application to a nonlinear problem

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We give the Strichartz estimates for the damped wave equation $\partial_t^2 \phi - \Delta \phi + \partial_t \phi = 0$. Moreover, we apply them to a nonlinear damped wave equation.

Blow-up and lifespan estimate to a nonlinear wave equation in Schwarzschild spacetime

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Luk (2013, Journal Eur. Math. Soc.) proved global existence for semilinear wave equations in Kerr spacetime with small angular momentum($a \ll M$)

$$\Box_{g_K} \phi = F(\partial \phi),$$

when the quadratic nonlinear term satisfies the null condition. In this work, we will show that if the null condition does not hold, at most we can have almost global existence for semilinear wave equations with quadratic nonlinear term in Schwarzschild spacetime, which is the special case of Kerr with $a = 0$

$$\Box_{g_S} \phi = (\partial_t \phi)^2,$$

where $\Box_{g_S}$ denotes the D'Alembert operator associated with Schwarzschild metric. What is more, if the power of the nonlinear term is replaced with p satisfying $3/2 \leq p < 2$, we still can show blow-up, no matter how small the initial data are. We do not have to assume that the support of the data should be far away from the event horizon.

On the scattering problem for the nonlinear Schrödinger equation with a potential in 2D

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This is a joint work with Vladimir Georgiev. We consider the scattering problem for the nonlinear Schrödinger equation with a potential in 2D. Appropriate resolvent estimates are proved and applied to estimate the operator $A(s)$ appearing in commutator relations. The equivalence between the operators $(-\Delta_V)^{\frac{s}{2}}$ and $(-\Delta)^{\frac{s}{2}}$ in L^2 norm sense for $0 \leq s < 1$ is investigated by using free resolvent estimates and Gaussian estimates for the heat kernel of the Schrödinger operator $-\Delta_V$. Our main result guarantees the global existence of solutions and time decay of the solutions assuming initial data have small weighted Sobolev norms. Moreover, the global solutions obtained in the main result scatter.

Asymptotic decay results for critical subelliptic equations

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In this talk we shall present some recent results on the asymptotic behavior of finite energy solutions to critical growth equations in the subelliptic setting. We deal with the semilinear case with Hardy perturbation and

the quasilinear case. The involved operators are sub-Laplacians on Carnot groups and Grushin-type operators. As a remarkable consequence of our analysis, we obtain the exact rate of decay of the extremal functions for some relevant functional inequalities on Stratified Lie groups, whose analytic expression is not known.

Energy estimate for wave equation with bounded time-dependent coefficient

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In this talk we present a good energy estimate for wave equation with time-dependent coefficient. To this end, we employ asymptotic integrations method, and investigate oscillation property of amplitude functions in representation formula of solutions. This talk is based on the joint work with Michael Ruzhansky (Ghent Univ. and Queen Mary Univ.).

Strong instability for standing wave solutions to the system of the quadratic NLKG

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We consider the instability for standing wave solutions to the system of the quadratic nonlinear Klein-Gordon equations. In the single case, namely the nonlinear Klein-Gordon equation with power type nonlinearity, stability and instability for standing wave solutions have been extensively studied. On the other hand, in the case of our system, there would be no result concerning the stability and instability for the standing wave as far as we know. In this talk, we give a strong instability result for the standing wave to our system. The proof is based on the techniques in Ohta and Todorova (2007). New ingredient is to need the mass resonance condition in two or three space dimensions whose cases are the mass sub-critical case.

Spectral theory for magnetic Schrödinger

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In this talk we present a new approach to growth estimates of generalized eigenfunctions for magnetic Schrödinger operators in exterior domain with exploding or oscillating potentials, apply them to show the principle of limiting absorption.

Fractional wave equation with discontinuous coefficients

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We consider wave propagation in viscoelastic media having non-constant density $\rho = \rho(x)$, obeying Zener constitutive law giving relation between stress and strain and with non-constant Young modulus of elasticity $E = E(x)$. If for ρ and E we allow also discontinuities, for example jumps modeling media consisting of two or more different materials, it would imply partial integro-differential equations with distributional coefficients. Existence of very week solution for such equation is going to be proved.

Work is done in collaboration with Michael Ruzhansky.

A new approach on the Calderón problem for discontinuous complex conductivities

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In this talk we present the inverse conductivity problem for complex conductivities with jumps. Such materials have never been considered in the literature where still the case of Lipschitz conductivities are assumed. For the study of this problem we model it as an interior transmission problem. To treat this problem several new concepts are required, such as an adaptation of the notion of scattering data, and the definition of admissible points, which permit the enlargement of the set of CGO incident waves. This will allow us to prove the reconstruction of the conductivity.
Given that this are early results in this direction, we also present some of the footwork necessary to proceed further in this direction.

Boundary value problem for the biharmonic operator in a ball

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This talk deals with several boundary value problems (bvp) for the biharmonic operator in the unit ball. They are divided into two different classes: (a) bvp satisfying Shapiro-Lopatinskii condition and (b) bvp violating the latter requirement. In the case (a) existence results of Fredholm type hold, while in the case (b) non-Fredholm results could appear. A bvp is of non-Fredholm type if it possesses infinite dimensional kernel or cokernel. For several examples explicit formulas for the corresponding solutions are given.

Local smoothing of Fourier integral operators and Hermite functions

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In this talk, we discuss a local smoothing estimate for Fourier integral operators of the form

$$\mathcal{F}f(x,t) = \int_{\mathbb{R}^2} e^{i(x\cdot\xi+t|\xi|)}\, a(x,t,\xi)\, \hat{f}(\xi)\, d\xi$$

for a wide class of symbols $a \in S^m(\mathbb{R}^2 \times \mathbb{R} \times \mathbb{R}^2)$, $m \leq 0$. Our result generalises the well known local smoothing estimate of Mockenhaupt, Seeger and Sogge appeared in 1992, to a global result with respect to the space variable. We use only a mild decay assumption on the amplitude function. The novelty in our approach is the use of Hermite functions in the study of Fourier integral operators. This is a joint work with Ramesh Manna.

Harmonic and Anharmonic Oscillators on the Heisenberg Group

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We propose a canonical version of the harmonic oscillator on the Heisenberg group $\mathbf{H}_n$ in terms of the representation theory of the Dynin-Folland group $\mathbf{H}_{n,2}$, a 3-step stratified Lie group, whose generic representations act on $L^2(\mathbf{H}_n)$. The classical relation between the harmonic oscillator on $\mathbb{R}^n$ and the sum of squares in the first stratum of the Heisenberg Lie algebra $\mathfrak{h}_n$ has an analog, which we turn into a definition: we define the harmonic oscillator on $\mathbf{H}_n$ as the image of the sub-Laplacian $\mathcal{L}_{\mathbf{H}_{n,2}}$ under the generic unitary irreducible representation π of the Dynin-Folland group which has formal dimension $d_\pi = 1$. More generally, this approach gives rise to a large class of so-called anharmonic oscillators by employing positive Rockland operators on $\mathbf{H}_{n,2}$. Employing methods developed by Elst and Robinson, we obtain spectral estimates for the harmonic and anharmonic oscillators on $\mathbf{H}_n$. The second part of the talk focuses on useful $L^{\mathbf{p}}$-$L^{\mathbf{q}}$-estimates for spectral multipliers of the sub-Laplacian $\mathcal{L}_{\mathbf{H}_{n,2}}$ and, more generally, of general Rockland operators on general graded groups. As a by-product, we recover Sobolev embeddings on graded groups, and obtain explicit hypoelliptic heat semigroup estimates. This is joint work with Prof. Michael Ruzhansky (Ghent University, Queen Mary University of London).

Geometric Hardy inequalities in the half-space on stratified groups

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In this talk, we present the geometric Hardy inequalities in the half-space on stratified groups, as a consequence we derive the geometric Hardy-Sobolev inequality on the Heisenberg group.

On a wave equation with singular dissipation

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We consider a wave equation with singular dissipation term and provide information on the behaviour of very weak solutions near the singularity.

Product formulas and convolutions for solutions of Sturm-Liouville equations

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The Fourier transform, which lies at the heart of the classical theory of harmonic analysis, is generated by the eigenfunction expansion of the Sturm-Liouville operator $-\frac{d^2}{dx^2}$. This naturally raises a question: *is it possible to generalize the main facts of harmonic analysis to integral transforms of Sturm-Liouville type?* In this talk we introduce a novel unified framework for the construction of product formulas and convolutions associated with a general class of regular and singular Sturm-Liouville boundary value problems. This unified approach is based on the application of the Sturm-Liouville spectral theory to the study of the associated hyperbolic equation. As a by-product, an existence and uniqueness theorem for degenerate hyperbolic Cauchy problems with initial data at a parabolic line is established. We will show that each Sturm-Liouville convolution gives rise to a Banach algebra structure in the space of finite Borel measures in which various probabilistic concepts and properties can be developed in analogy with the classical theory. We will discuss whether the convolution structure satisfies the basic axioms of the theory of hypergroups. Examples will be given, showing that many known convolution-type operators — such as the Hankel, Jacobi and Whittaker convolutions — can be constructed using this general approach.

A trial to construct specific self-similar solutions for nonlinear wave equations

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An attempt to construct self-similar solutions to nonlinear wave equations $\Box u = |u|^p$ and $\Box u = |u|^{p-1}u$ will be explained. The existence of self-similar solutions has been already established by Pecher (2000), Kato-Ozawa (2003), etc, based on the standard fixed point theorem. In this talk, we will discuss it by a constructive method based on a relation to the hypergeometric differential equation.

Local well-posedness for higher order Benjamin-Ono type equations

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In this talk, we consider the local well-posedness for higher order Benjamin-Ono type equations, especially fourth order equations. The proof is based on the energy method with correction terms. Our equations have at most three derivatives in nonlinear terms, so that we need to cancel out all derivative losses by introducing correction terms into the energy. We also employ the Bona-Smith approximation technique in order to show the continuity of the flow and the continuous dependence.

Endpoint Strichartz estimates for Schrödinger equation on exterior domains

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In this talk we consider the initial-boundary value problem of Schrödinger equation with Dirichlet or Neumann boundary condition on the exterior domain of a geometric non-trapping obstacle. Our goal is to prove endpoint Strichartz estimates for solutions to the problem. For this purpose, we start by introducing suitable local extension operators, and then combine these extension operators with Strichartz and smoothing estimates for the free Schrödinger equation, the known local smoothing estimates for the exterior boundary value problem, and some commutator estimates between polynomial weights and fractional differential operators. This talk is based on the joint work with Professor Vladimir Georgiev (University of Pisa).

Decay and Scattering in energy space for the solution of generalised Hartree equation

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We prove decay with respect some Lebesgue norms for a class of Schrödinger equations with non-local nonlinearities by showing new Morawetz inequalities and estimates. As a straightforward product we obtain large-data scattering in the energy space for the solutions to the defocusing generalized Hartree equations with mass-energy intercritical nonlinearities in any space dimensions.

Very weak solutions of wave equation for Landau Hamiltonian with irregular electromagnetic field

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In this paper, we study the Cauchy problem for the Landau Hamiltonian wave equation, with the time-dependent irregular (distributional) electromagnetic field and similarly irregular velocity. For such equations, we describe the notion of a 'very weak solution' adapted to the type of solutions that exist for regular coefficients. The construction is based on considering Friedrichs–type mollifier of the coefficients and corresponding classical solutions, and their quantitative behaviour in the regularising parameter. We show that even for distributional coefficients, the Cauchy problem does have a very weak solution, and that this notion leads to classical or distributional type solutions under conditions when such solutions also exist.

Blowing-up solutions of the time-fractional dispersive partial differential equations

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This paper is devoted to the study of initial-boundary value problems for time-fractional analogues of Korteweg-de Vries-Benjamin-Bona-Mahony-Burgers, Rosenau-Korteweg-de Vries-Benjamin-Bona-Mahony-Burgers, Ostrovsky and time-fractional modified Korteweg-de Vries-Burgers equations on a bounded domain. Sufficient conditions for the blowing-up of solutions in finite time of aforementioned equations are presented. We also discuss the maximum principle and in uence of gradient non-linearity on the global solvability of initial-boundary value problems for the time-fractional Burgers equation. The main tool of our study is the Pohozhaev nonlinear capacity method. We also provide some illustrative examples.

Inverse Scattering Problems on Quantum Graphs

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I investigate inverse scattering problems for a Sturm-Liouvile operator on the metric graph consisting of a finite number of half-lines joined with either a loop or a finite number of finite intervals.
The scattering matrix, part of the negative eigenvalues and corresponding normalizing coefficients are taken as a scattering data.
The main goal of this research is to reconstruct the coefficients of Sturm-Liouvile operator on the basis of the given scattering data. We have deduced Marchenko equation which allowed us to prove the uniqueness theorems, provided a reconstruction procedure for the coefficients on the half-lines and investigated the conditional stability of the inverse scattering problem.
Later on I have used the asymptotic behaviour of the scattering matrix to investigate the geometry of the graph.

Optimal symmetric range for Hilbert transform and its non-commutative extensions

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We identify the optimal range of the Calderòn operator and that of the classical Hilbert transform in the class of symmetric quasi-Banach spaces. Further consequences of our approach concern the optimal range of the triangular truncation operator, of operator Lipschitz functions and commutator estimates in ideals of compact operators.

Local uniqueness of ground states for generalized Choquard model

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We consider the generalized Choquard equation describing trapped electron gas in 3 dimensional case. The study of orbital stability of the energy minimizers (known as ground states) depends essentially in the local uniqueness of these minimizers. In equivalent way one can optimize the Gagliardo–Nirenberg inequality subject to the constraint fixing the L^2 norm. The uniqueness of the minimizers for the case $p = 2$, i.e. for the case of Hartree–Choquard is well known. The main difficulty for the case $p > 2$ is connected with possible lack of control on the L^p norm of the minimizers.

Index transforms with the squares of Kelvin functions

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New index transforms, involving squares of Kelvin functions, are investigated. Mapping properties and inversion formulas are established for these transforms in Lebesgue spaces. The results are applied to solve a boundary value problem on the wedge for a fourth order partial differential equation.

Hypoelliptic functional inequalities and applications

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In this talk we will give a review of our recent research on hypoelliptic functional inequalities. We managed to link the integral versions of Hardy inequalities on homogeneous groups to their hypoelliptic versions through the Riesz and Bessel kernels of the Rockland operators (hypoelliptic left-invariant homogeneous differential operators, following the Helffer-Nourrigat's resolution of the Rockland conjecture in the 80s). Consequently, this leads to general hypoelliptic versions of Hardy-Sobolev, Hardy-Littlewood-Sobolev, Trudinger-Moser, Caffarelli-Kohn-Nirenberg, Gagliardi-Nirenberg and other inequalities (https://arxiv.org/abs/1805.01064). We will then concentrate also on discussing their best constants, ground states for higher order hypoelliptic Schrödinger type equations, and the solutions to the corresponding variational problems (https://arxiv.org/abs/1704.01490).

Global regularity for Einstein-Klein-Gordon system with $U(1) \times \mathbb{R}$ isometry group

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In this paper we prove the global regularity of the 3+1 dimensional Einstein-Klein Gordon system with a $U(1) \times \mathbb{R}$ isometry group. We reduce the Cauchy problem of the Einstein-Klein Gordon system to a 2+1 dimensional system. We prove that the energy of this system cannot concentrate near the first possible singularity, and hence is small. Then, we show that the global regularity holds for the reduced 2+1 system with initial data of small energy.

S.13 Integral Transforms and Reproducing Kernels

Organisers
Zouhaïr Mouayn

Scope of the session: Over the years, motivated by applications in initial and boundary value ordinary and partial differential equations, many integral transforms have been constructed, leading to rich mathematical theories with an outstanding effectiveness in physical and signal analytic applications. A notorious famous example is the use of the Fourier transform in heat conduction problems and signal analysis of band-limited functions. Today, a number of transforms have reached a classical status, namely those associated with the names of Laplace, Fourier, Mellin, Hankel, Radon and Hilbert. They are all widely used in physics and signal analysis. More recently, many fractional integral transforms became popular in image reconstruction, pattern recognition and acoustic signal processing. A notable class of integral transforms, developed with the goal of understanding non-stationary phenomena in several geometries is provided by the so-called coherent states transforms CST (including Gabor and wavelet transforms, by specifying the underlying geometries to euclidean and hyperbolic and the groups structure to the Heisenberg and affine cases). They have been used in Schroedinger equations with specific potential, in the analysis of higher Landau level eigenspaces, in the analysis of nonstationary signals, in analytic and polyanalytic function theory, in frame theory and, more recently, in determinantal point processes (for instance, in Ginibre-type models for higher Landau levels and in the so-called Weyl-Heisenberg ensemble). On the background of all these applications is the fact that the range of CST is a reproducing kernel Hilbert space. Another useful transform defined via reproducing kernels is the Berezin transform, which shows up naturally in quantization-dequantization procedures. Our aim in this session is

to discuss old and new integral transforms: their origin, their mathematical properties (in particular, of their reproducing kernel structure) and their potential use to solve problems in mathematics, signal analysis and physics.

—Abstracts—

Models and classifications in fractional calculus

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Fractional calculus is a branch of analysis in which one can differentiate or integrate functions a non-integer number of times. Generalising the basic operations of calculus to non-integer orders allows a wider range of behaviours to be captured. As it turns out, the range is very wide indeed, because there is no unique way to define fractional derivatives and integrals: a whole host of different possible models have been proposed. This has led to several different points of view within the field.
From the mathematical perspective, the key idea is *generalisation*. The proliferation of definitions of fractional calculus gives rise to many unique challenges and properties which are not found in classical calculus. For example, most fractional derivatives are nonlocal and do not satisfy a semigroup law. There is also a desire for *definition*: what exactly makes a particular operator a fractional derivative or not?
From the applications perspective, the key idea is *modelling*. Many processes which arise in real life can be more accurately modelled using fractional operators – of various types – than using classical calculus. For example, viscoelastic materials may be described by fractional differential equations with order between those of the equations describing viscosity and elasticity. Any definition of fractional calculus which is useful physically is worth analysing mathematically.
After summarising the different approaches found within the fractional calculus community, I will attempt to unify them by introducing some broad *classes* of fractional operators. Especially important is the class defined by a convolution integral with an analytic function of a fractional power: this is broad enough to cover many different types of fractional calculus, yet still clearly connected to the classical Riemann–Liouville fractional calculus. It can be studied on a general level, as well as divided into subclasses with specific desired properties.

Paley-Wiener and Wiener's Tauberian results for a class of oscillatory integral operators

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The main aim of this talk is to obtain Paley–Wiener and Wiener's Tauberian results associated with a class of oscillatory integral operators, which depends on cosine and sine kernels, as well as to introduce a consequent new convolution associated with that class of operators. Additionally, a new Young-type inequality for the obtained convolution is proven, and a new Wiener-type algebra is also associated with this convolution. This is based on a joint work with L. P. Castro and N. M. Tuan.

On the boundedness of composition operators on reproducing kernel Hilbert spaces with analytic positive definite functions

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In this talk, I would like to explain our result that boundedness of composition operators of maps implies the maps are affine maps in certain situations. Composition operators (Koopman operators) are classically investegated in complex analysis, but, they have been getting popular in the context of machine leraning and data analysis these days. Besides, reproducing kernel Hilbert spaces with analytic positive definite functions on euclidean spaces are utilized in many fields in engeneering and statistics. On the other hand, although it is important to prove the relation between the properties of maps and those of composition operators of the maps to guruantee theoretical validity, such relation is currently not known very well. In some important situation, we prove that a map become an affine map if its composition operator is bounded on an RKHS associated with analytic positive definite functions on euclidean spaces. This is the joint work with Masahiro Ikeda (RIKEN/Keio University) and Yoshihiro Sawano (Tokyo metropolitan University/RKEN).

A harmonic interpolation sequence on the real unit ball

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We provide sufficient conditions for interpolating a sequence by a function in a weighted Bergman type space of harmonic functions on the real unit ball.

Nonlinear coherent states associated with a measure on the positive real half line

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We construct a class of generalized nonlinear coherent states by means of a newly obtained class of 2D complex orthogonal polynomials. The associated coherent states transform is discussed. A polynomials realization of the basis of the quantum states Hilbert space is also given. Here, the entire structure owes its existence to a certain measure on the positive real half line, of finite total mass, together with all its moments. We illustrate this construction with the example of the measure $r^{\beta}e^{-r}dr$, which leads to a new generalization of the true-polyanalytic Bargmann transform.

Distributed-order non-local optimal control

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Distributed-order fractional non-local operators have been introduced and studied by Caputo at the end of the 20th century. They generalize fractional order derivatives/integrals in the sense that such operators are defined by a weighted integral of different orders of differentiation over a certain range. The subject of distributed-order non-local derivatives is currently under strong development due to its applications in modeling some complex real world phenomena. Fractional optimal control theory deals with the optimization of a performance index functional subject to a fractional control system. One of the most important results in classical and fractional optimal control is the Pontryagin Maximum Principle, which gives a necessary optimality condition that every solution to the optimization problem must verify. In our work, we extend the fractional optimal control theory by considering dynamical systems constraints depending on distributed-order fractional derivatives. Precisely, we prove a weak version of Pontryagin's maximum principle and a sufficient optimality condition under appropriate convexity assumptions.

Convolution integral equations related to Fourier sine and cosine transforms and Hermite functions

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In this talk, we present new convolutions on the positive half-line for Lebesgue integrable functions. Factorization identities for those convolutions are derived, upon the use of Fourier sine and cosine transforms and Hermite polynomials. These results allow us to consider (systems of) convolution integral equations, to analyse conditions for their solvability and, under such conditions, to obtain their solutions.

Noetherian Solvability of an Operator Singular Integral Equation with Carleman Shift in Fractional Spaces

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Let Γ be a Lyapunov closed contour of class $C^1_\nu, \frac{2}{p}-1<\nu\leq 1, 1<p<2$. We consider in Besov's space $B(\Gamma)\equiv B^r_{p,1}(\Gamma), r=\frac{1}{p}, 1<p<2$,the singular integral equation

$$L\varphi\equiv a(t)\varphi(t)+b(t)\varphi[\alpha(t)]+\frac{c(t)}{\pi i}\int_\Gamma\frac{\varphi(\tau)}{\tau-t}d\tau$$

$$+\frac{d(t)}{\pi i}\int_\Gamma\frac{\varphi(\tau)}{\tau-\alpha(t)}d\tau+\int_\Gamma K(t,\tau)\varphi(\tau)d\tau=g(t)$$

where $a(t), b(t), c(t), d(t) and g(t)$ belong to space $B(\Gamma), \alpha(t)$ is Carleman's shift, i.e. homeomorphically maps Γ onto itself, preserving or changing its orientation to Γ and $\alpha[\alpha(t)]=t$. It is supposed, that there exists a derivative $\alpha'(t)$, belonging to space $H_\mu(\Gamma)$ of Holder continuous functions with indicator $\frac{2}{p}-1<\nu\leq 1$.The kernels $K_j(t,\tau), j=1,2$, have such weak singularities that the corresponding integral operators are completely continuous in $B(\Gamma)$. We note, that $B(\Gamma)$ embedded in the class of continuous functions $C(\Gamma)$, but not embedded in $H_\mu(\Gamma) 0<\mu\leq 1$, is a commutative Banach algebra with a unit with the usual operations of addition and multiplication of functions [1, 2]. Equation (1) is considered in [3] in spaces $H_\mu(\Gamma) 0<\mu\leq 1$, and $L_p(\Gamma), 1<p<\infty$. We have obtained the conditions of the noetherian solvability of the equation (1) in $B(\Gamma)$ and its index formula. The results are used [4].

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Joint talk with Bliev N.K., bliev.nazarbay@mail.ru

S.14 Partial Differential Equations on Curved Spacetimes

Organisers
Anahit Galstyan, Makoto Nakamura, Karen Yagdjian

Scope of the session: The session will discuss various problems for partial differential equations on curved spacetimes. The topics will include the representation and qualitative properties of the solutions of initial value and boundary value problems for linear and nonlinear PDEs in the curved spacetimes, global existence and blow up of the solutions, and numerical simulations, etc.

—Abstracts—

Computational analysis of a nonlinear wave equation with black hole embedded in an expanding universe

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Numerical simulations are presented for a nonlinear wave equation with singularity that models black hole and with time dependent coefficient that models an expanding universe. The high performance Pycuda computations use an explicit fourth order Runge-Kutta scheme on the temporal discretization and fourth order finite difference discretization in the 3-dimensional space. Bubble formation and the properties of solutions

with compact support are examined both inside and outside of the blackhole.

Radiation fields for wave equations

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Radiation fields are rescaled limits of solutions of wave equations near "null infinity" and capture the radiation pattern seen by a distant observer. They are intimately connected with the Fourier and Radon transforms and with scattering theory. In this talk, I will define and discuss radiation fields in a few contexts, with an emphasis on spacetimes that look flat near infinity. The main result is a connection between the asymptotic behavior of the radiation field and a family of quantum objects on an associated asymptotically hyperbolic space. This talk is based on joint work with Jeremy Marzuola, Andras Vasy, and Jared Wunsch

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Local existence of solutions to the Euler–Poisson system, including densities without compact support

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Local existence and uniqueness for a class of solutions for the Euler Poisson system is shown. These solutions have a density ρ which either falls off at infinity or has compact support. Since the Euler equations degenerate in such a setting, a regularisation has to be considered which complicates the proof considerably. The solutions have finite mass, finite energy functional and include the static spherical solutions for $\gamma = \frac{6}{5}$. The result is achieved by using weighted Sobolev spaces of fractional order and a new non linear estimate which allows to estimate the physical density by the regularised non linear matter variable. With these tools at hand we then prove the existence of solutions to the Euler–Poisson–Makino system by using a fixed-point argument. Gamblin also has studied this setting but using very different functional spaces. However, we believe that the functional setting we use is more appropriate to describe a physical isolated body and more suitable to study the Newtonian limit. The talk is based on a collaboration with L. Karp.

About critical exponents in semi-linear de Sitter models

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In this paper we consider the Cauchy problem for semilinear de Sitter models. The model of interest is

$$\phi_{tt} - e^{-2t}\Delta\phi + n\phi_t + m^2\phi = F_p(\phi),$$
$$(\phi(0,x), \phi_t(0,x)) = (f(x), g(x)),$$

where n is the dimension, m^2 is a non-negative constant and p is a positive parameter. Our main goal is to verify that, in general, one can not observe a critical exponent $p_{crit} = p_{crit}(n)$ in the family of non-linearities $\{F_p(\phi)\}_{p>1} = \{|\phi|^p\}_{p>1}$. Moreover, we like to propose parameter-dependent families $\{F_p(\phi)\}_{p>0}$ of non-linear right-hand sides where the correct choice of the parameter p might provide a threshold between global (in time) existence of small data solutions (stability of the zero solution) and blow-up behavior even of small data solutions.

Semilinear Klein-Gordon Equation in the Friedmann-Lamaitre-Robertson-Walker spacetime

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We present a condition on the self-interaction term that guaranties the existence of the global in time solution of the Cauchy problem for the semilinear Klein-Gordon equation in the Friedmann-Lamaitre-Robertson-Walker (FLRW) model of the contracting universe. For the Klein-Gordon equation with the Higgs potential we give a lower estimate for the lifespan of solution.

Heat Flow of Isometric G_2-structures

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Given a 7-dimensional compact Riemannian manifold (M, g) that admits G_2-structure, all the G_2-structures that are compatible with the metric g are parametrized by unit sections of an octonion bundle over M. We define a natural energy functional on unit octonion sections and consider its associated heat flow. The critical points of this functional and flow precisely correspond to G_2-structures with divergence-free torsion. In this talk, we first derive estimates for derivatives of $V(t)$ along the flow and prove that the flow exists as long as the torsion remains bounded. We will also show a monotonicity formula and and an epsilon-regularity result for this flow. Finally, we show that within a metric class of G_2-structures that contains a torsion-free G_2-structure, under certain conditions, the flow will converge to a torsion-free G_2-structure.

Energy estimates for Klein-Gordon type equations with time dependent mass

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We consider the energy estimate of the solution to the Cauchy problem of Klein-Gordon type equation with time dependent mass:

$$\begin{cases} \partial_t^2 u - \Delta u + M(t)u = 0, & (t,x) \in (0,\infty) \times \mathbb{R}^n, \\ u(0,x) = u_0(x), \ \ \partial_t u(0,x) = u_1(x), & x \in \mathbb{R}^n, \end{cases}$$

where $M(t)$ is real valued, oscillating and not necessarily positive. The main purpose of my talk is to give sufficient conditions to $M(t)$ for the energy to be asymptotically stable.

Continuous dependence on the geometrical initial data for the Einstein vacuum equations

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The Cauchy problem of Einstein equations deals with the construction of a Lorenztian manifold from given geometric quantities on an initial sub manifold. The solutions consists of two major steps, namely, solution of the constraint and evolution equations. The constraint equations can be written as an elliptic system on the initial manifold and their solutions provide the initial data for the evolution equations. Under the harmonic gauge the evolution equations are reduced to quasilinear wave equations. The common methods to solve these central problems in asymptotically flat spacetime is to consider the constraint equations in a weighted Sobolev spaces, while the quasilinear wave equation are dealt in the ordinary unweighted Sobolev spaces. Therefore it is impossible to obtain well–posedness of these equations by this approach. We treat both central type of equations in the weighted Sobolev spaces, and hence we are able to derive the well–posedness of the Cauchy problem for Einstein equations in asymptotically flat spacetime, including continuous depending of the Lorentzian metric on the initial geometrical data.
The talk is based on a joint work with U. Brauer.

Asymptotic Behavior of nonlinear Schrödinger equations with radial data

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The asympototic behavior of solutions to dispersive equations is a very important question. The problem is particularly difficult if the equation admits solitons. In this talk, we will report some of our recent progress in this direction for nonlinear Schrödinger equations with radial data.

Remarks on the Navier-Stokes equations in homogeneous and isotropic spacetimes

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The Navier-Stokes equations are considered in homogeneous and isotropic spacetimes. The energy estimates of the equations are constructed. The effects of the spatial expansion and contraction are characterized by the dissipative and anti-dissipative properties of the equations. The Cauchy problem for the Navier-Stokes equations is considered under the constant density.

Solutions of the wave equation bounded at the Big Bang

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By solving a singular initial value problem, we prove the existence of solutions of the wave equation $\Box_g \phi = 0$ which are bounded at the Big Bang in the Friedmann-Lemaître-Robertson-Walker cosmological models. More precisely, we show that given any function $A \in H^3(\Sigma)$ (where $\Sigma = \mathbb{R}^n, \mathbb{S}^n$ or $\mathbb{H}^n$ models the spatial hypersurfaces) there exists a unique solution ϕ of the wave equation converging to A in $H^1(\Sigma)$ at the Big Bang, and whose time derivative is suitably controlled in $L^2(\Sigma)$.

Semilinear de Sitter model of cosmology - global existence of small data solutions

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In this talk we consider the Cauchy problem for semilinear de Sitter models with power non-linearity. The model of interest is

$$\phi_{tt} - e^{-2t}\Delta\phi + n\phi_t + m^2\phi = |\phi|^p,$$
$$(\phi(0,x), \phi_t(0,x)) = (f(x), g(x)),$$

where m^2 is a non-negative constant. We study the global (in time) existence of small data solutions. In particular, we show the interplay between the power p, admissible data spaces and admissible spaces of solutions (in weak sense, in sense of energy solutions or in classical sense).
These are joint considerations with Marcelo Rempel Ebert (University of Sao Paulo).

Outgoing Fredholm theory and the limiting absorption principle for asymptotically conic spaces

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In this talk I will discuss geometric generalizations of Euclidean resolvent estimates, such as estimates for the resolvent of the Laplacian of an asymptotically conic metric plus a decaying potential, in a Fredholm framework that focuses on capturing the outgoing asymptotics of the resolvent applied to a Schwartz function (outgoing waves); this is different from even the usual treatment of the Euclidean problem. More precisely, the setting is that of perturbations $P(\sigma)$ of the spectral family of the Laplacian $\Delta_g - \sigma^2$ on asymptotically conic spaces (X, g) of dimension at least 3 (with the asymptotic behavior

at the 'large end' of the cone), and the main results are the limiting absorption principle, as well as uniform estimates for $P(\sigma)^{-1}$ as $\sigma \to 0$, on function spaces between which $P(\sigma)$ is Fredholm even for real $\sigma \neq 0$ and which correspond to finite regularity Lagrangian distributions associated to the, conic in the base, Lagrangian given by the outgoing radial set of the Hamilton flow. Such results have immediate applications to the behavior of the wave equation on black hole spacetimes.

Quantum Field Theory on Low regularity Spacetimes

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In this talk we develop the theory of quantum fields on low regularity curved spacetimes. We follow the algebraic approach as outlined in the book by Bär, Ginoux and Pfäffle suitably adapted to the low regularity setting. In particular we will show how to obtain a unique global $H^2_{loc}(M)$ causal solution to the Cauchy problem for a normally hyperbolic operator on a globally hyperbolic $C^{1,1}$ spacetime. We then show how to construct advanced and retarded Green operators on suitable function spaces and use this to construct the causal propagator G. This is then used to define a symplectic form ω on $H^1_{comp}(M)/Ker(G)$. Finally this is used to construct a representation of the canonical commutation relations on the space of quasi-local C^*-algebras which satisfy the Haag-Kastler axioms. Defining the physical states requires a Sobolev space variant of the micro-local spectrum condition of Radzikowski.

On the Cauchy problem for hyperbolic operators with triple characteristics whose coefficients depend only on the time variable

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We have considered the Cauchy problem for hyperbolic operators with double characteristics whose principal symbols have time-dependent coefficients and given sufficient conditions and necessary conditions for C^∞ well-posednesss, before. Here we deal with the Cauchy problem for hyperbolic operators with triple characteristics whose coefficients are real analytic functions of the time variable. We fatorize the operators as products of hyperbolic operators whose symbols are polynomials of τ of degree ≤ 3, where τ is the dual variable of the time variable. For hyperbolic operators of third order we defined the sub-sub-principal symbols and we proved the Cauchy is C^∞ well-posed imposing conditions on the subprincipal symbol and the sub-sub-principal symbol. We shall generalize the conditions for operators of third order and give the sufficient conditions that the Cauchy problem for hyperbolic operators with triple characteristics is C^∞ well-posed. For special operators of third order we shall prove our conditions are necessary for C^∞ well-posedness.

Navier-Stokes equation with very rough data

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We study the Cauchy problem for the incompressible Navier-Stokes equation (NS):

$$u_t - \Delta u + u \cdot \nabla u + \nabla p = 0, \quad \operatorname{div} u = 0, \quad u(0,x) = u_0.$$

We consider a class of very rough initial data in $E^s_{2,2}$ for which the norm are defined by

$$\|u_0\|_{E^s} = \|2^{s|\xi|}\widehat{u}_0(\xi)\|_{L^2}, \quad s < 0$$

and show that NS has a unique global solution if the initial value $u_0 \in E^s$, $s < 0$ and their Fourier transforms are supported in $\mathbb{R}^d_I := \{\xi \in \mathbb{R}^d : \ \xi_i \geq 0, i = 1, ..., d\}$. Our results imply that NS has a unique global solution if the initial value u_0 is in L^2 with $\operatorname{supp} \widehat{u}_0 \subset \mathbb{R}^d_I$. This is a joint work with Professors Feichtinger, Gröchenig and Dr Li.

Properties of solutions of hyperbolic equations in the curved space-times

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In this talk we describe some quantitative and qualitative properties of the solutions of hyperbolic equations in the curved space-times. This properties are obtained by representation formulas written via the integral transform approach that was developed by the author. Special attention is given to the solutions of the linear and semilinear equations in the FLRW space-times.

Global solutions of massive Maxwell-Klein-Gordon equations with large Maxwell field

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In this talk, I will discuss the global asymptotic dynamics for solutions of massive Maxwell-Klein-Gordon equations with a class of unrestricted large data, which have arbitrary size of Maxwell field. The main difficulties are arising from the largeness of the Maxwell field and the different decay properties for linear Maxwell field and linear massive Klein-Gordon equation, which will cause a loss of decay for solutions of the coupled system at the causal infinity within a forward light cone. The first issue can be addressed by using the modified vector field method developed by Klainerman-Wang-Yang while the loss of decay is overcome by uncovering a hidden null structure.

S.15 Partial Differential Equations with Nonstandard Growth

Organisers
Hermenegildo Borges de Oliveira,
Sergey Shmarev

Scope of the session: The aim of this Session is to discuss the theoretical and numerical aspects of the analysis of Partial Differential Equations with nonstandard growth. This class of equations embraces the equations whose nonlinear structure may vary in the space/time domain and is allowed to depend on the independent variables or even in the solution itself. During the last decades, PDEs of this type have been the focus of attention of many researchers. On the one hand, the interest in the study of such equations is explained by their applications in the mathematical modelling of the real world processes, in particular, in the models of fluid dynamics or in processing of digital images. On the other hand, their theoretical study leads to challenging problems related to the issues of existence, uniqueness and qualitative properties of the solutions.

—Abstracts—

On a multidimensional boundary value problem for a model degenerate elliptic-parabolic equation

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For a degenerate elliptic-parabolic equation boundary value problems are mainly studied in the two-dimensional case. The main bibliographies on these problems are given in the works of Salakhitdinov M.S., Dzhuraev T.D., Muratbekov M.B. and Kalmenov T.Sh. In the present work, by using the methods of spectral decomposition, it is established the unique solvability of the classical multidimensional problem of a degenerate elliptic-parabolic equation. In this case, the boundary conditions are defined only in the domain of the ellipticity of the equation and also proved smoothness of solutions in the elliptic and parabolic domains.

Interior Regularity to the Steady Incompressible Shear Thinning Fluids with Non-standard Growth

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We consider weak solutions to the equations of stationary motion of a class of non-Newtonian fluids which includes the power law model. The power depends on the spatial variable, which is motivated by electrorheological fluids. We prove the existence of second order derivatives of weak solutions in the shear thinning cases

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Well-posedness of the Cosserat-Bingham multi-dimensional fluid equations

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We consider a model for Cosserat-Bingham fluids. In contrast to the classical Bingham fluid, the Cosserat-Bingham fluids support local micro-rotations and two types of plug zones. We show the global-in-time solvability of the initial boundary value problem for the Cosserat-Bingham model.
Joint work with A. de Araujo (UFV, Brasil), M.Santos (UNICAMP, Brasil).

Boundary value problem involving $p-$Laplacian and jumping nonlinearities

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We get one theorem which shows existence of solutions for boundary value problem involving p-Laplacian and jumping nonlinearities. This theorem is that there exists at least one solution when nonlinearities crossing finite number of eigenvalues. We obtain this result by the eigenvalues and the corresponding normalized eigenfunctions of the $p-$Laplacian eigenvalue problem, variational reduction method and critical point theory.

Optimal feedback control of second grade fluids

Fernanda Cipriano
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This talk deals with a feedback optimal control problem for the stochastic second grade fluids. More precisely, we establish the existence of an optimal feedback control for the two-dimensional stochastic second grade fluids, with Navier-slip boundary conditions. In addition, using the Galerkin approximations, we show that the optimal cost can be approximated by a sequence of finite dimensional optimal costs. This is a joint work with Diogo Pereira.

On the numerical simulation of a parabolic equation with p-Laplacian and memory.

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In this talk we consider the following parabolic equation with the p-Laplacian and a memory term in a one dimensional spatial domain:

$$u_t(x,t) = (|u_x(x,t)|^{p-2}u_x(x,t))_x + \int_0^t g(t-s)(|u_x(x,s)|^{p-2}u_x(x,s))_x\,ds + f(x,t).$$

The main numerical issues concern the accurate estimation of the p-Laplacian and the computer memory consumption. For the spatial discretization, we consider a

continuous piecewise polynomial finite element approximation. In order to discretize the time variable we propose the Crank-Nicolson method with the trapezoidal rule to handle the memory term. Finally, several simulations addressing some properties of the solutions are presented.
This is a joint work with Rui Almeida and Belchior Mário.

Picone identity for variable exponents operators and applications

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The talk is concerned with new results about Diaz-Saa type inequality and Picone identity for quasilinear elliptic operators with variable exponents. Various applications to uniqueness and global behaviour for quasilinear elliptic and parabolic problems are derived.

About some generalizations of the parabolic p-Laplacian and the Porous Medium Equation

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In this talk we will present and discuss some properties of two generalizations of the two well known parabolic PDEs, the parabolic p-Laplacian and the porous medium equation, namely

$$u_t - \nabla \cdot \left(u^{\gamma(x,t)} \nabla u \right) = 0,$$

where $-1 < \gamma^- \leq \gamma(x,t) \leq \gamma^+ < \infty$ and $\gamma \in L^\infty \left(0, T; W^{1,p}(\Omega)\right)$, for some $p > \max\{2, N\}$ and

$$\partial_t(u^q) - \nabla \cdot \left(|\nabla u|^{p-2} \nabla u \right) = 0, \quad 0 < q, \quad p < 1.$$

Multiplicity results for $p-$Laplacian boundary value problem

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We investigate multiplicity of solutions for one dimensional $p-$Laplacian Dirichlet boundary value problem with jumping nonlinearites. We obtain three theorems: The first one is that there exists exactly one solution when nonlinearities cross no eigenvalue. The second one is that there exist exactly two solutions, exactly one solutions and no solution depending on the source term when nonlinearities cross one first eigenvalue. The third one is that there exist at least three solutions, exactly one solutions and no solution depending on the source term when nonlinearities cross the first and second eigenvalues. We obtain the first theorem and the second one by eigenvalues and the corresponding normalized eigenfunctions of the $p-$Laplacian eigenvalue problem, and the contraction mapping principle on $p-$Lebesgue space (when $p \geq 2$). We obtain the third result by Leray-Schauder degree theory.

A free boundary problem for an inhomogeneous operator with nonstandard growth

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We present recent results on a free boundary problem for an inhomogeneous operator with nonstandard growth. This operator has been used in the modelling of electrorheological fluids and in image processing. We obtain free boundary regularity results for weak solutions and we apply these results to minimizers of suitable energy functionals. In our work we overcome deep technical difficulties not present in previous literature for this type of problems. This is joint work with Noemi Wolanski (Universidad de Buenos Aires and IMAS - CONICET).

About a new non linear boundary condition for the turbulent kinetic energy involved in models of turbulence

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We first derive from a standard modeling process a new non linear boundary condition for the turbulent kinetic energy k, that is autonomous in k. We then show that the corresponding model of turbulence, coupling k to the mean velocity and the mean pressure of a turbulent flows, has a distributional solution. This analysis is illustrated by several numerical simulations in a 3D channel that shows the accuracy of the model.

On a nonlocal viscosity p-Laplacian evolutive problem. Existence and long-time behavior

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We analyze the (global) existence of solutions for the scalar evolutive p-Laplacian problem with nonlocal viscosity and extra terms (eventually another nonlinearity and a time-dependent) in the right hand side

$$\begin{cases} \dfrac{du}{dt} - a(l(u))\Delta_p u = f(u) + h(t) & \text{in } \Omega \times (\tau, T), \\ u = 0 \quad \text{on } \partial\Omega \times (\tau, T), \\ u(x,\tau) = u_\tau(x) \quad \text{in } \Omega, \end{cases}$$

where Ω is a bounded open set of $\mathbb{R}^N$, $p \geq 2$, $a \in C(\mathbb{R}; [m, \infty))$, $m > 0$, and $l(u) = \int_\Omega g(x)u(x)dx$ for a given $g \in L^2(\Omega)$.
The non-standard nonlocal diffusion term given by the function a aims to model both quick aggregation or oppositely, the necessity to leave crowded zones, depending on the choice of the viscosity function, non-increasing or increasing. To better model certain situations with rough viscosity terms only continuity is assumed for the viscosity function.
These two nonlinearities (nonlocal term and p-Laplacian) combined with the fact of an evolutive problem involves extra difficulties to solve it, which are circumvented by a nontrivial change of variables, compactness and monotonicity techniques. The regularizing effect of the equation is shown by applying an argument of a posteriori regularity, even although uniqueness of solution for the problem is unknown.
Besides that, the long-time behavior of solutions is analyzed via multi-valued dynamical systems and the existence of several families of (pullback) attractors ensured, in $L^2(\Omega)$, $L^p(\Omega)$ and actually in any Banach space X such that the following compact/continuous embeddings $W_0^{1,p}(\Omega) \subset\subset X \subset L^2(\Omega)$ hold.
This is a joint work with Tomás Caraballo and Marta Herrera-Cobos (Universidad de Sevilla).

A survey of recent results on the characterization of decay of solutions to some dissipative equations

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Solutions to many dissipative equations in Fluid Mechanics obey inequalities which show that their energy decays in time. In this talk we will describe recent results in which a complete characterization of decay rates is obtained through a single number associated to the initial datum, for a large family of equations including Navier-Stokes and dissipative quasi-geostrophic.

Existence results for the $p(u)-$Laplacian problem.

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In this talk, we consider the $p-$Laplacian problem with the exponent of nonlinearity p depending on the solution u itself. For the associated boundary-value and initial boundary-value problems, we prove the existence of weak solutions by using a singular perturbation technique. We will also point out some challenging problems in this class of partial differential equations with nonstandard growth. This talk is based in joint works in collaboration with Michel Chipot from Zrich Universitt.

Entropy and kinetic solutions of the genuinely-nonlinear Kolmogorov-type equation with partial diffusion and nonlinear source term

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The talk is devoted to a study of the Cauchy–Dirichlet problem for quasilinear Kolmogorov-type ultra-parabolic equation incorporating two time-like variables and a nonlinear source term:

$$(1a)\quad \partial_t u + \partial_s a(u) + \operatorname{div}_x \boldsymbol{\varphi}(u) = \Delta_x u + h_\gamma(\boldsymbol{x}, t, s, u), \quad \text{where } (\boldsymbol{x}, t, s) \in G_{T,S},$$
$$(1b)\quad u|_{t=0} = u_0^{(1)},\ u|_{s=0} \approx u_0^{(2)},\ u|_{s=S} \approx u_S^{(2)},\ u|_\Omega = 0.$$

Here $G_{T,S} := \Omega \times (0,T) \times (0,S)$; Ω is a bounded domain of spatial variables $\boldsymbol{x} \in \mathbb{R}^d$ with a smooth boundary $\partial\Omega$; $t \in [0,T]$ and $s \in [0,S]$ are two independent time-like variables, $T, S = \text{const} > 0$; $u = u(\boldsymbol{x}, t, s)$ is a sought function; initial and final data $u_0^{(1)}$, $u_0^{(2)}$ and $u_S^{(2)}$, nonlinearities a, $\varphi_1, \ldots, \varphi_d$ and source term h_γ are given and smooth. Function a satisfies the special *genuine nonlinearity condition*:

$$(2)\quad \text{the set } \{\lambda \in \mathbb{R} :\ \xi_1 + a'(\lambda)\xi_2 = 0\} \text{ has the empty interior for each fixed } (\xi_1, \xi_2) \in \mathbb{S}^1.$$

In general, function a is non-monotone, which means that evolution process in time-like direction s may be reversible. Condition (2) is the cornerstone of the proper entropy and kinetic formulations of problem (1a)-(1b). Justification of existence and uniqueness of entropy and kinetic solutions is the first major result of this study. The second major result relates with the case when h_γ collapses to the Dirac delta-function as $\gamma \to 0$. We show that there is a limiting point $u = \lim\limits_{\gamma \to 0} u_\gamma$ of the family of solutions to problem (1a)-(1b) such that u solves the *impulsive* equation of the form (1a), with the sudden source term $\delta_{(t=\tau)}\beta(\boldsymbol{x}, s, u)$ on the place of $h_\gamma(\boldsymbol{x}, t, s, u)$.
The presented research was fulfilled in collaboration with ***Dr Ivan Kuznetsov*** (Novosibirsk State University).

Solvability of Nonlinear Elliptic Type Equation With Two Unrelated Non standard Growths

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In this paper, we study the solvability of the nonlinear Dirichlet problem with sum of the operators of independent non standard growths in a bounded domain $\Omega \subset \mathbb{R}^n$. Here, one of the operators in the sum is monotone and the other is weakly compact. We obtain sufficient conditions and show the existence of weak solutions of the problem under consideration by using monotonicity and compactness methods together.

Global regularity of solutions of singular parabolic equations with nonstandard growth

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We present new results on the global regularity properties of solutions to parabolic equations which involve $p(x)$ and $p(x,t)$-Laplace operators. The solutions of such equations are usually understood in a weak sense. In particular, the time derivative is a distribution which does not belong to any Lebesgue space. We find conditions on the data that guarantee the existence of strong solutions. For these solutions, the second derivatives in space and the first derivative in time belong to Lebesgue spaces with variable exponents prompted by the equation. It is shown that under certain conditions on the data the solutions of the parabolic problem are continuous with respect to time in the sense of Hölder and Lipschitz. This is joint work with Prof. S.Antontsev.

On a stochastic $p(\omega,t,x)$-Laplace equation

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A stochastic forcing of a non-linear singular/degenerated parabolic problem of $p(\omega,t,x)$-Laplace type is proposed in the framework of Orlicz-Lebesgue and Sobolev spaces with variable random exponents. We give a result of existence and uniqueness of the solution, for additive and multiplicative problems.
This is joint work with Guy Vallet and Aleksandra Zimmermann.

Sufficient conditions for local regularity to the generalized Newtonian fluid with shear thinning viscosity

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We prove the local regularity of a weak solution u to the equations of a generalized Newtonian fluid with power law $1 < q < 2$ if u belongs to a suitable Lebesgue space. This result extends the well-known Serrin condition for weak solutions of the Navier-Stokes equations to the shear-thinning fluids.

S.16 Pseudo Differential Operators

Organisers
Shahla Molahajloo, Man Wah Wong

Scope of the session: This is intended to be a special session in the mathematics of pseudo-differential operators to be interpreted in a very broad sense. Topics include functional analysis, operator theory and operators, harmonic analysis, partial differential equations, applications and computation related to pseudo-differential operators.

—Abstracts—

Two-wavelet curvelet localization operators and two-wavelet curvelet multipliers

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The main aim of this talk is to introduce and study the two-wavelet curvelet localization operators and the two-wavelet curvelet multipliers.
To this end we give a resolution of the identity formula for high frequency signals (in fact for images) in $L^2_{2/a_0,\pi-\sqrt{a_0}}(\mathbb{R}^2) = \{f \in L^2(\mathbb{R}^2); \hat{f}(\xi) = 0$, if $|\xi| < 2/a_0$ or $|arg\,\xi| > \pi - \sqrt{a_0}\}$ for some $a_0 \in (0,\pi^2)$ and another one for arbitrarily signals f in $L^2(\mathbb{R}^2)$.
Let $X = (0,a_0) \times \mathbb{R}^2 \times [-\pi,\pi]$ be the measure space equipped with the measure $d\mu = \frac{da}{a^3} dbd\theta$. Then in the following two theorems we give the two resolution identity formulae mentioned above.
Theorem 1. Let $f \in L^2_{2/a_0,\pi-\sqrt{a_0}}(\mathbb{R}^2)$. Then $(f,g) = \int_X (f,\gamma_{ab\theta})(\tilde{\gamma}_{ab\theta},g)d\mu$, for all $g \in L^2(\mathbb{R}^2)$, where $\gamma_{ab\theta}, \tilde{\gamma}_{ab\theta} : \mathbb{R}^2 \to \mathbb{C}$ are functions defined by means of radial windows $W, \tilde{W}$ and angular windows $V, \tilde{V}$, which have some suitable properties.
Theorem 2. For all $f,g \in L^2(\mathbb{R}^2)$, $(f,g) = \int_{\mathbb{R}^2} (f,\Phi_b)(\tilde{\Phi}_b,g) + \int_X (f,\gamma_{ab\theta})(\tilde{\gamma}_{ab\theta},g)d\mu$, where Φ and $\tilde{\Phi}$ are father wavelets, which satisfies some conditions and for all $b \in \mathbb{R}^2$ we defined $\Phi_b(x) = \Phi(x-b)$, $x \in \mathbb{R}^2$.
In the following we give some boundedness results concerning the two-wavelet curvelet localization operators $L_\tau : L^2_{2/a_0,\pi-\sqrt{a_0}}(\mathbb{R}^2) \to L^2(\mathbb{R}^2)$ defined by $(T_\tau f,g) = \int_X \tau(a,b,\theta)(f,\gamma_{ab\theta})(\tilde{\gamma}_{ab\theta},g)d\mu$ for all $f \in L^2_{2/a_0,\pi-\sqrt{a_0}}(\mathbb{R}^2)$ and $g \in L^2(\mathbb{R}^2)$, which have their symbols τ in $L^\infty(X) \cup L^1(X)$. We also state and prove some boundedness results referring to the two-wavelet curvelet multipliers $T_\sigma : L^2(\mathbb{R}^2) \to L^2(\mathbb{R}^2)$ defined by $(T_\sigma f,g) = \int_{\mathbb{R}^2} \sigma(b)(f,\Phi_b)(\tilde{\Phi}_b,g)db$, for all f and g in $L^2(\mathbb{R}^2)$ with their symbols σ in $L^\infty(\mathbb{R}^2) \cup L^1(\mathbb{R}^2)$. In addition some results concerning the trace class property and the trace of the two-wavelet curvelet multipliers are given.

Pseudo-differential operators of Gevrey type and their action on classes of modulation spaces

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We deduce one-parameter group properties for pseudo-differential operators $\mathrm{op}(a)$, where a belongs to symbol classes of Gevrey type, associated with suitable, non-vanishing, weight functions ω_0, and denoted by $\Gamma_*^{(\omega_0)}$. This allows to show that there are pseudo-differential operators $\mathrm{op}(a)$ and $\mathrm{op}(b)$ which are inverses to each others, and satisfy $a \in \Gamma_*^{(\omega_0)}$ and $b \in \Gamma_*^{(1/\omega_0)}$.
We apply these results to deduce lifting properties for classes of modulation spaces and construct isomorpisms between them. In particular, for any couple of admissible weight functions ω, ω_0, we prove that the Toeplitz operator (or localization operator) $\mathrm{Tp}(\omega_0)$ is an isomorphism from the weighted modulation space $M^{p,q}_{(\omega)}$

to the weighted modulation space $M^{p,q}_{(\omega/\omega_0)}$, for every $p, q \in (0, \infty]$.
This is joint work with A. Abdeljawad and J. Toft.

Pseudo-Differential Operators and Existence of Gabor Frames

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In this talk we present, from a a pseudo-differential point of view, the behavior of the frame operator associated with a Gabor system. In particular we show how an application of the classical boundedness theorem of Calderón -Vaillancourt yields sufficient conditions for a Gabor system to form a frame in $L^2(\mathbb{R}^d)$.

Continuity of linear operators on mixed-norm Lebesgue and Sobolev spaces

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Mixed-norm Lebesgue spaces found their place in the study of some questions in the theory of partial differential equations, as it can be seen from recent interest in continuity of certain classes of pseudodifferential operators on these spaces. We present a general framework for dealing with continuity of linear operators on these spaces. This allows us to prove the boundedness of a large class of pseudodifferential operators, and also the boundedness of integral operators on mixed-norm Lebesgue spaces. In some cases, the generalisations to mixed-norm Sobolev spaces are obtained as well, together with applications to some interpolation and compactness results.

C^*-algebras, H^*-algebras and trace ideals of pseudo-differential operators on locally compact, Hausdorff and abelian groups

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In this talk, we define pseudo-differential operators on a locally compact, Hausdorff and abelian group G as natural extensions of pseudo-differential operators on $\mathbb{R}^n$. In particular, for pseudo-differential operators with symbols in $L^2(G \times \widehat{G})$, where $\widehat{G}$ is the dual group of G, we give explicit formulas for the products and adjoints, characterize them as Hilbert–Schmidt operators on $L^2(G)$ and prove that they form a C^*-algebra, which is also a H^*-algebra. We give a characterization of trace class pseudo-differential operators in terms of symbols lying in a subspace of $L^1(G \times \widehat{G}) \cap L^2(G \times \widehat{G})$.

This is a joint work with Prof. M. W. Wong (York University).

Pseudos with limited smoothness applied to strictly hyperbolic Cauchy problems with coefficients low-regular in time and space

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Pseudodifferential operators are a well-known tool to study smooth hyperbolic Cauchy problems. Even if the coefficients are low-regular in time, we are still able to obtain well-posedness results by using classical or weighted symbol spaces and the related operators.
In this talk, however, we consider strictly hyperbolic Cauchy problems with coefficients low-regular in t and x. By this we mean that the coefficients have a modulus of continuity below Lipschitz in t and belong to the Zygmund space C^*_s in x. Our goal is to make s as small as possible and still have an energy estimate with no loss of derivatives.
To prove an energy estimate for this problem, we apply pseudodifferential operators with limited smoothness in x. We discuss how results for mapping properties, adjoints and sharp Gårding's inequality influence our final result. The talk is concluded by some examples.

Paley-Wiener Theorems of real type in ultradifferentiable spaces

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In this talk we present some kind of "real Paley-Wiener theorems" for classes of rapidly decreasing ultradifferentiable functions. The word "real" expresses that information about the support of f comes from growth rates associated to the function $\hat{f}$ on R^d rather than on C^d as in the classical (complex) Paley-Wiener theorems. This theory was initiated by Bang and Tuan, and here we follow the approach of Andersen and De Jeu, who prove characterizations, relating support to growth, of the image of the Schwartz spaces of rapidly decreasing functions under the Fourier transform. In this talk we consider as functional setting the space S_ω of ultradifferentiable functions of Beurling type and investigate the relations between the support of the Fourier transform of an ultradifferentiable function f and the behaviour of f, its derivatives, and its Wigner transform. We introduce results of this type for the so-called Gabor transform and give a full characterization in terms of Fourier and Wigner transforms for several variables of a Paley-Wiener theorem in this general setting.

Fredholm Property of Non-Smooth Pseudodifferential Operators

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Hence great effort already was spent to get some conditions for the Fredholmness of smooth pseudodifferential operators with symbols in the Hörmander-class

$S^m_{\rho,\delta}(\mathbb{R}^n \times \mathbb{R}^n)$. Finally Schrohe was able to show that the uniformly ellipticity of certain smooth symbols is a necessary and sufficient condition for the Fredholmness of the associated pseudodifferential operator considered as a map between certain weighted Bessel potential spaces. In applications also non-smooth pseudodifferential operators occur. The goal of this talk is to give sufficient condiditions for the Fredholm property of non-smooth pseudodifferential operators with symbols in the class $C^\tau S^0_{1,0}(\mathbb{R}^n \times \mathbb{R}^n; N)$.
The talk is based on a joint work with H. Abels.

Analytic pseudo-differential calculus via the Bargmann transform

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The Bargmann transform is a transform which maps Fourier-invariant function spaces and their duals to certain spaces of formal power series expansions, which sometimes are convenient classes of analytic functions. In the 70th, Berezin used the Bargmann transform to translate problems in operator theory into a pseudo-differential calculi, where the involved symbols are analytic functions, and the corresponding operators map suitable classes of entire functions into other classes of entire functions. Recently, some investigations on certain Fourier invariant subspaces of the Schwartz space and their dual (distribution) spaces have been performed by the author. These spaces are called Pilipović spaces, and are defined by imposing suitable boundaries on the Hermite coefficients of the involved functions or distributions. The family of Pilipović spaces contains all Fourier invariant Gelfand-Shilov spaces as well as other spaces which are strictly smaller than any Fourier invariant non-trivial Gelfand-Shilov space. In the same way, the family of Pilipović distribution spaces contains spaces which are strictly larger than any Fourier invariant Gelfand-Shilov distribution space. In the talk we show that the Bargmann images of Pilipović spaces and their distribution spaces are convenient classes of analytic functions or power series expansions which are suitable when investigating analytic pseudo-differential operators. We also deduce continuity properties for such pseudo-differential operators when the symbols and target functions possess certain (weighted) Lebesgue estimates. We also show that the counter image with respect to the Bargmann transform of these results generalise some continuity results for (real) pseudo-differential operators with symbols in modulation spaces, when acting on other modulation space. The talk is based on collaborations with *Nenad Teofanov* from the University of Novi Sad, and the content of the talk is available at

N. Teofanov, J. Toft *Pseudo-differential calculus in a Bargmann setting*, Ann. Acad. Sci. Fenn. Math. (to appear).

Time-frequency analysis and pseudo-differential operators on locally compact groups

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Time-frequency analysis can be described as Fourier analysis simultaneously both in time and in frequency. Its origins are in quantum mechanics, in signal processing in Euclidean spaces, and in pseudo-differential operators. In this presentation we show how to generalize time-frequency analysis to those locally compact groups that allow a nice-enough Fourier transform: wide families of non-commutative groups can be treated. Our results on locally commutative groups shed new light also on the analysis in Euclidean spaces.

Defect distributions applied to differential equations with polynomial coefficients

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H-distributions were introduced by Antonić and Mitrović (2011) for weakly convergent sequences in dual pair of $L^p - L^q$ spaces.
Defect distributions are extension of H-distributions. They are connected with weakly convergent sequences in weighted Sobolev spaces $H^{s,p}_\Lambda(\mathbb{R}^d)$. We prove existence of defect distributions using weighted type classes of symbols and global pseudo-differential calculus. Defect distributions related with elliptic symbols give us connection between weakly and strongly convergent sequences in appropriate Sobolev spaces.
We apply results in order to analyze equations with polynomial coefficients. We consider sequence of approximate equations

$$\sum_{(\alpha,\beta)\in V(\mathcal{P})} x^\beta D^\alpha u_n(x) = f_n(x)$$

and obtain information about the set of points where weakly convergent sequence u_n is (locally) strongly convergent.
This is joint work with Stevan Pilipović.

Strong continuity on Shubin-Sobolev spaces of semigroups for evolution equations of quadratic operators

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We consider the initial value Cauchy problem for evolution equations of Schrödinger type defined by a Hamiltonian that is the Weyl quantization of a quadratic form with nonnegative real part. We show that the corresponding solution semigroup is strongly continuous on Shubin-Sobolev spaces of integer order. This gives uniqueness and continuity of solutions when the initial datum is a tempered distribution.

Normality, Self-Adjointness, Spectral Invariance, Groups and Determinants of Pseudo-Differential Operators on Finite Abelian Groups

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We give the normality, self-adjointness and spectral invariance of pseudo-differential operators on finite abelian groups. We also give a formula for the determinant of every element in a group of pseudo-differential operators on a finite abelian group.

S.17 Quaternionic and Clifford Analysis

Organisers
Swanhild Bernstein, Uwe Kähler,
Irene Sabadini, Franciscus Sommen

Scope of the session: Quaternionic and Clifford analysis are higher dimensional analogues of complex analysis and represent a function theory in $\mathbb{R}^n$. In the last years quaternionic and Clifford analysis spread into several different directions such as continuous and discrete theory, hermitian Clifford analysis, higher spin operators, slice monogenic functions as well as new applications to various fields. This session aims to present recent advances in the field of continuous and Clifford analysis as well as its applications in numerical analysis of PDE's, signal and image processing, operator theory, and physics to a broad audience.

—Abstracts—

A general setting for functions of Fueter variables: differentiability, rational functions, Fock module and related topics

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We develop some aspects of the theory of hyperholomorphic functions whose values are taken in a Banach algebra over a field – assumed to be the real or the complex numbers – and which contains the field. Notably, we consider Fueter expansions, Gleason's problem, the theory of hyperholomorphic rational functions, modules of Fueter series, and related problems. Such a framework includes many familiar algebras as particular cases. The quaternions, the split quaternions, the Clifford algebras, the ternary algebra, and the Grassmann algebra are a few examples of them.
This is joint work with Ismael Paiva and Daniele Struppa.

Sufficient Conditions for Associated Operators to a Space of Harmonic-Type functions

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In this talk, we deal with the second-order differential operator $\tilde{\Delta} = div(B\nabla)$ and the first-order differential operator $\mathcal{F} = \sum_{i=0}^{n} a^{(i)}(x)\partial_{x_i}u(x) + b(x)u(x) + c(x)$, where B is a symmetric and positive definite matrix in $\mathbb{R}^{n+1\times n+1}$, $x = (x_0, x_1, \ldots, x_n) \in \mathbb{R}^{n+1}$, $a^{(i)}, u \in C^2(\mathbb{R}^{n+1}; \mathcal{A}_n)$, $b, c \in C^1(\mathbb{R}^{n+1}; \mathcal{A}_n)$ and $\mathcal{A}_n$ is the classic Clifford algebra with structure relations

$$e_i e_j + e_j e_i = -2\delta_{ij}, \quad i, j = 1, 2, \ldots, n.$$

We give sufficient conditions for operators $\tilde{\Delta}$ and $\mathcal{F}$ to be associated. We also explore the necessary conditions and, provided a bounded domain Ω is given, we exhibit an interior estimate for harmonic functions in the supremum norm. This is could be used to study the existence and uniqueness of solutions of some particular initial value problems.

A Dirac operator in infinite dimensional analysis

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We aim to construct a Dirac operator in the setting of infinite dimensional analysis. Let H be a real Hilbert space and $\mu = N_Q$ a non-degenerate Gaussian measure on H. We denote by (ξ_k) a complete orthonormal system in H and by (λ_k) a sequence of positive numbers such that $Q\xi_k = \lambda_k\xi_k$ and by $\mathfrak{E}(H)$ the space of all exponential functions. For any $\varphi \in \mathfrak{E}(H)$ and any $k \in \mathbb{N}$ then

$$D_k\varphi(x) = \lim_{\varepsilon\to 0} \frac{1}{\varepsilon}\left[\varphi(x + \varepsilon\xi_k) - \varphi(x)\right], \quad x \in H$$

and the Dirac operator is defined as

$$D = \sum_{k=0}^{\infty} e_k D_k,$$

where (e_k) is a sequence of generating vectors of an infinite dimensional Clifford algebra such that $e_i e_j + e_j e_i = \delta_{ij}$.
The mapping

$$D : (E) \subset L^2(H, \mu) \to L^2(H, \mu; H), \quad \varphi \mapsto D\varphi$$

is closeable. Moreover, $D^2 = -\Delta_G$, the Gross Laplacian.

On Runge Pairs and Topology of Axially Symmetric Domains.

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Subsequently to the very interesting and seminal paper of Colombo, Sabadini and Struppa in 2011, we will present a new theorem on Runge Pairs of Axially Symmetric Domains in the quaternionic setting, that can be

easily generalized to the Clifford environment. A *global approach* (in which we strongly believe) and a *complete study of the homology groups* of these domains, instead of a slicewise analysis, have permitted us to improve the results in Colombo, Sabadini and Struppa's original paper and a more deep understanding of the subject. This is part of a joint project with J. Winkelmann.

Special classes of monogenic functions and applications in linear elastic fracture mechanics

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Recently, the classical orthogonal function systems of inner and outer monogenic Appell functions were used to find new classes of monogenic functions with (logarithmic) line singularities, which extend the known function classes in a natural way. In the talk we will discuss some essential properties (orthogonality, Appell property, three-term recurrence relation) of the classical and the special classes of monogenic functions as well as their relations to each other. Furthermore, on the basis of the generalized Kolosov-Muskhelishvili formulas, it will be shown how these special functions can be used to construct analytical near-field solutions of spatial crack fronts in linear elastic fracture mechanics.

Interpolation by polynomials over division rings

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Polynomials over a division ring $\mathbb{F}$ can be evaluated "from the left" and "from the right" giving rise to left and right interpolation problems. If the interpolation problem involves interpolation conditions of the same (left or right) type, the results are similar to the commutative case: a consistent problem has a unique solution of a low degree (less than the number of interpolation conditions imposed), and the solution set of the homogeneous problem is an ideal of $\mathbb{F}[z]$. The problem containing both "left" and "right" interpolation conditions is quite different: there may exist infinitely many low-degree solutions and the solution set of the homogeneous problem is a quasi-ideal in $\mathbb{F}[z]$. In the talk, we will discuss the two-sided Lagrange interpolation formula, two-sided polynomial independence, and extensions of polynomials within a given conjugacy class.

GASPT, Radially Holomorphic Functions, the Fueter Potential Method in an Inhomogeneous Medium and Problems of Generalized Joukowski Transformations in $\mathbb{R}^3$

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The Fueter potential method in the setting of GASPT is provided. An axially symmetric generalization of the Cauchy-Riemann system in $\mathbb{R}^3 = \{(x_0, x_1, x_2)\}$, including the longitudinal variable x_0 and the cylindrical radial variable $\rho = \sqrt{x_1^2 + x_2^2}$, and further the Cauchy-Riemann system in the meridian plane, spanned by (x_0, ρ), play the main roles. New classes of Fueter mappings of the first kind and the second kind are defined. Corresponding classes of irrotational meridional velocity fields in an axially symmetric inhomogeneous medium are described as gradient dynamical systems with variable dissipation. Problems of the sets of critical points of the scalar velocity potential and problems of singular sets of spatial axially symmetric generalizations of conformal mappings of the second kind are studied. Fueter mappings of the second kind, characterized by the Hessian matrix J, allow us to develop important applications of radially holomorphic functions $u = u_0 + Iu_\rho = u(x)$, where $x = x_0 + I\rho$. Herewith $I = \frac{ix_1 + jx_2}{\rho}$, $Iu_\rho = iu_1 + ju_2$ and $i^2 = j^2 = I^2 = -1$. The Fueter holomorphic primitive $U = U_0 + IU_\rho = U(x)$ of a radially holomorphic function $u = u(x)$, satisfying conditions $U' = \frac{\partial U}{\partial x_0} = u$, where $IU_\rho = iU_1 + jU_2$, plays a key role of Fueter potential in continuum mechanics. In particular, properties of models of generalized Joukowski transformations of order n in the form $U(x) = \frac{1}{2}(x^n + x^{-n})$ are described. The Fueter potential method assumes three principal invariants I_J, II_J, III_J and three real roots λ_1, λ_2, λ_3 of the corresponding characteristic equation. **Theorem**. Roots of the characteristic equation $\lambda^3 - I_J\lambda^2 + II_J\lambda - III_J = 0$ in the framework of the Fueter potential method are described by formulas:

$$\lambda_1 = \frac{u_\rho}{\rho}, \qquad \lambda_{2,3} = \pm\sqrt{\left(\frac{\partial u_\rho}{\partial x_0}\right)^2 + \left(\frac{\partial u_\rho}{\partial \rho}\right)^2}.$$

Some applications of the Spectral Theory on the S-spectrum

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In this talk we give an overview of the spectral theory based on the notion of S-spectrum. This theory turned out very useful to define new classes of fractional diffusion and fractional evolution processes. We will show new results on the quaternionic version of the H^∞-functional calculus and we use it to compute the fractional powers of vector operators. Such fractional powers of generalized gradient operators define non-local Fourier's? laws for the propagation of the heat.

A right inverse operator for curl $+ \lambda I$ and applications

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In this talk will be presented a general solution of the equation

$$curl\overrightarrow{w} + \lambda\overrightarrow{w} = \overrightarrow{g},$$

where λ is an arbitrary non-zero complex number and $\overrightarrow{g}$ belongs to the class of L^p-integrable functions whose divergence is also L^p-integrable. In other words, a right inverse operator for the operator $curl+\lambda I$ is constructed in this class of integrable functions. The explicit general solution is based on the use of classical integral operators of quaternionic analysis as well as on the construction of metaharmonic conjugate functions. Applications of the above result are considered to the nonhomogeneous time-harmonic Maxwell system.

On the Bargmann-Fock-Fueter and Bergman-Fueter integral transforms (Joint work with Prof. Krausshar and Prof. Sabadini)

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The classical complex Fock space and associated Segal-Bargmann transform define some important mathematical models used in quantum mechanics. In this talk, we present some special integral transforms of Bargmann-Fock type in the setting of quaternions. Our constructions are based on the well-known Fueter mapping theorem. In particular, starting with the normalized Hermite functions we construct an Appell system of quaternionic regular polynomials. The ranges of such integral transforms are quaternionic reproducing kernel Hilbert spaces of regular functions. Some new integral representations and generating functions in this setting are obtained in both the Fock and Bergman cases.

Sufficient Conditions for the Fractional Vekua equation to be Associated with the fractional Cauchy-Riemann operator in the Riemann-Liouville sense

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Consider the fractional Cauchy-Riemann operators $D_+^{\alpha,\beta} = D_{x_0^+}^{\alpha} + iD_{x_1^+}^{\beta}$ in the Riemann-Liouville sense. Where $\overline{D_+^{\alpha,\beta}}$ is its conjugate. With this operator, we can write the fractional Vekua equation in the Riemann-Liouville sense:

$$\partial_t\omega = A\overline{D_+^{\alpha,\beta}}\omega + BD_+^{\alpha,\beta}\omega + CD_+^{\alpha,\beta}\overline{\omega} + E\overline{D_+^{\alpha,\beta}}\overline{\omega} + F\omega + G\overline{\omega} + H = \Psi\omega.$$

where A, B, C, E, F, G and H are complex valued functions and t is the time. Ψ is a linear first order operator acting in the complex plane.
On the other hand. A pair (L, l) of fractional differential operators is said to be associated if $l(u) = 0$ implies $l(Lu) = 0$.
In this work we formulate sufficient conditions on the coefficients of operator $\Psi\omega$ under which this operator is associated to the fractional Cauchy-Riemann operator in the Riemann-Liouville sense, using a matrix representations (this is: if $D_+^{\alpha,\beta}\omega = 0$ then $D_+^{\alpha,\beta}(\Psi\omega) = 0$).

Hyperbolic function theory and hyperbolic Brownian motion

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We study hyperbolic function theory of quaternion valued functions depending on three coordinates. A functions is called α-hyperbolic harmonic if it is harmonic with respect to the Riemannian metric

$$ds_k^2 = \frac{dx_0^2 + dx_1^2 + dx_2^2}{x_2^{2\alpha}}$$

in the upper half space $\mathbb{R}^3_+ = \{(x_0, x_1, x_2,) \in \mathbb{R}^3 : x_2 > 0\}$. An interesting result is that a real valued function f is α-hyperbolic harmonic if and only if the function $g(x) = x_3^{\frac{1-\alpha}{2}} f(x)$ is the eigenfunction of the hyperbolic Laplace operator $\triangle_h = x_2^2\triangle - x_2\frac{\partial}{\partial x_2}$ corresponding to the eigenvalue $\frac{1}{4}\left((\alpha+1)^2 - 4\right) = 0$. This means that in case $\alpha = 1$, the 1-hyperbolic harmonic functions are harmonic with respect to the hyperbolic metric of the Poincaré upper half-space. Leutwiler started to study this case around 1990 and noticed that the quaternionic power function x^m $(m \in \mathbb{Z})$, is a conjugate gradient of a 1-hyperbolic harmonic function.
The theory was generalized to total quaternion valued functions by defining modified Dirac operators and α-hypermonogenic functions. Recently, we verified a Cauchy type integral formula with more concrete kernels for a-hypermonogenic functions. In this talk, we are presenting some connections of hyperbolic function theory to the hyperbolic Brownian motion. Hyperbolic Brownian motion is defined in terms of stochastic differential equations. The hyperbolic Brownian motion is a canonical diffusion on hyperbolic spaces and its generator is the half of the hyperbolic Laplace-Beltrami operator.

The discrete Cauchy-Kovalevskaya extension originating from fractional central difference operators

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On this talk we will report on recently ongoing results related to discrete time-changed fractional equations on the space-time lattice $h\mathbb{Z}^n \times (1-\alpha)\tau\mathbb{Z}$ of the form

$$\mathbf{e}_0\left(\frac{\sinh((1-\alpha)\tau\partial_t)}{(1-\alpha)\tau}\right)^{\beta}\Psi(x,t) - \mathbf{e}_0\phi_{\alpha,\beta}(t;\tau)\,\Phi_0(x)$$
$$= -\theta(2\alpha-\beta)\,D_h\Psi(x,t) - \mathbf{e}_{2n+1}\frac{\beta-2\alpha}{2\beta}\,\tau^{\beta}\Delta_h\Psi(x,t)$$

$(\mathbf{e}_0^2 = -1,\ \mathbf{e}_{2n+1}^2 = +1\ \&\ 0 \le \alpha, \beta < 1)$, carrying the fractional central difference operator $\left(\frac{\sinh((1-\alpha)\tau\partial_t)}{(1-\alpha)\tau}\right)^{\beta}$ and two given discretizations D_h resp. Δ_h of the Dirac and Laplace operator on the lattice $h\mathbb{Z}^n$ satisfying the factorization property $(D_h)^2 = -\Delta_h$.

The functions $\theta(2\alpha-\beta)$ resp. $\phi_{\alpha,\beta}(t;\tau)$ correspond to the *Heaviside step function* and to the fractional regularization of the *sinc function* on the limit $\beta\to\alpha^+$:

$$\lim_{\beta\to\alpha^+}\phi_{\alpha,\beta}(t;\tau)=(1-\alpha)\tau\frac{\sin\left(\frac{\pi t}{(1-\alpha)\tau}\right)}{\pi t}.$$

At a first glance, we will show that the solution of the above equation seamlessly describes the solution associated to the discrete counterpart of the Cauchy-Kovalevskaya extension.
Secondly, we will employ Fourier analysis techniques on the space-time toroidal manifold $\mathbb{R}^n/\frac{2\pi}{h}\mathbb{Z}^n\times\mathbb{R}/\frac{2\pi}{\tau(1-\alpha)}\mathbb{Z}$ to represent $\left(\frac{\sinh((1-\alpha)\tau\partial_t)}{(1-\alpha)\tau}\right)^\beta$ as a discrete convolution operator and to obtain a representation formula for the aforementioned equation. In case where $\phi_{\alpha,\beta}(t;\tau)$ equals to

$$\frac{\cos\left(\frac{\pi t}{(1-\alpha)\tau}\right)\Gamma(\beta-\alpha+1)}{\Gamma\left(\frac{\beta-\alpha+1}{2}+\frac{t}{(1-\alpha)\tau}+1\right)\Gamma\left(\frac{\beta-\alpha-1}{2}-\frac{t}{(1-\alpha)\tau}+1\right)}$$

one will also show that the underlying solution may be represented my means of Fourier-Laplace multipliers, carrying the generalized Mittag-Leffler function $E_{\beta,\alpha}(z)$. Special emphasize will be also given to the discrete Dirac semigroup $\{\exp(\mathbf{e}_0tD_h)\}_{t\geq0}$ ($2\alpha-\beta\geq0$ & $\tau\to0^+$) and to the cases where the above equation resembles to a stochastic process of Wiener type in the limit $\alpha\to1^-$.

Real Paley-Wiener Theorem for Clifford-Fourier Transfrom

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In the framework of Clifford analysis, we consider several versions of the real Paley-Wiener theorems for a generalized Clifford-Fourier transform. This Clifford-Fourier transform is given by a similar operator exponential as the classical Fourier transform but containing generators of Lie superalgebra. In addition, some related results for a fractional version of Clifford-Fourier transform are also presented at the end of this talk.

THE commutative diagram of signal processing from a Clifford perspective

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The commutative diagram of signal processing consists of the Fourier Series, the Paley-Wiener theorem and the Sampling Theorem. In the L^2-setting, the theory is well known, for the classical case as well as for the case of nonharmonic Fourier series and entire functions of exponential type with various growth conditions. For $L^p(\mathbb{R})$-spaces, $p\neq2$, the commutative diagram was established by Maergoiz in 2006. For entire functions with polygonal indicator diagram partial results are given by Levin/Ljubarski in 1975 and Semmler/F. in 2015. Recently, the Clifford community developed Paley-Wiener theorems for multivariate functions on Clifford-Hilbert-spaces, see e.g. Kou/Tian 2002 and Franklin/Hogan/Larkin 2017. In the presentation, we show the developments of THE commutative diagram with respect of these aspects and give first results for the Clifford setting. This is research in progress in collaboration with Jeff Hogan, University of Newcastle, Australia.

A Clifford Construction of Multidimensional Prolate Spheroidal Wave Functions

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We review the one dimensional prolate spheroidal wave functions (PSWFs) including their construction and importance as bandlimited functions. We then, recall the higher dimension version of PSWFs, the radial parts of which are the eigenfunctions of the operator

$$M_c(u)(t)=(1-t^2)\frac{d^{2u}}{dt}-2t\frac{du}{dt}+(-c^2t^2+\frac{\frac{1}{4}-N^2}{t^2})u.$$

which has a singularity at the origin, causing instabilities. We investigate the construction of multidimensional PSWFs using techniques from Clifford analysis. The Clifford-Legendre differential equation is given by

$$\partial_x^2C^0_{n,m}(P_k)(x)-2x\partial_xC^0_{n,m}(P_k)(x)$$
$$-C(0,n,m,k)C^0_{n,m}(P_k)(x)=0,$$

where ∂_x is Dirac operator on $\mathbb{R}^m$. We will find the Bonnet formula for the Clifford-Legendre polynomials and then we define the following operator

$$L_cf(x)=\partial_x((1-|x|^2)\partial_xf(x))+4\pi^2c^2|x|^2f(x),$$

which is Hermitian. We refer the eigenfunctions of the latter operator as "**Clifford multidimensional PSWFs.**" By the use of the Bonnet formula, we will try to compute the Clifford multidimensional PSWFs. Then we will prove that they are also the eigenfunctions of a time-frequency limiting operator in $L^2(B(1),\mathbb{R}_2)$. We will define the time-frequency limiting operator $\mathcal{G}_c$ by

$$\mathcal{G}_cf(x)=\chi_{B(1)}(x)\int_{B(1)}e^{2\pi ic\langle x,y\rangle}f(y)dy,$$

and show that L_c and $\mathcal{G}_c$ commute, a phenomenon known as the **"lucky accident."**

Slice regular functions in several variables

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In this talk we present basic concepts and results of a new function theory in several hypercomplex variables, including quaternions, octonions and Clifford algebras. This new function theory have been obtained jointly with Alessandro Perotti (University of Trento).

Compressed Sensing with Slice Monogenic Signals

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Here we propose the reconstruction of signals by basis-Pursuit, i.e., by ℓ_1-minimization and the sampling model is random points with high probability by using the slice regular monogenic functions.

Radial integration operators in infinite domains and their applications in quaternion analysis

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Radial integration operators I^α in bounded star-shaped domains are the effective tool in the mathematical physics and quaternion analysis. The first three-dimensional analogue of the Kolosov-Muskhelishvili formulae was obtained by means of such operators when a general solution of the Lame equation for the spatial theory of elasticity was expressed in terms of two regular quaternionic functions. In this report we introduce new radial integration operators J^α acting in an infinite space with a star-shaped cavity. It is shown that operators J^α have properties similar to I^α. By means of these operators J^α some problems of quaternion analysis are solved. Three-dimensional quaternionic analogue of the Kolosov-Muskhelishvili formulae in infinite space with a star-shaped cavity is constructed. As an application, the boundary value problem of equilibrium of infinite elastic space with spherical cavity is solved in a closed form.

A distributional approach to integration over smooth surfaces of lower dimension embedded in $\mathbb{R}^m$

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Methods of integration have been amply developed for parametrically defined $(m-k)$-surfaces in $\mathbb{R}^m$, i.e. defined as the image of a map from a subset of $\mathbb{R}^{m-k}$ to $\mathbb{R}^m$. In this case, the use of differential forms plays an important role. However, some methods of calculation are also necessary in the case where an $(m-k)$-surface is implicitly defined by means of k equations $P_1(x) = \cdots = P_k(x) = 0$. In this talk, we illustrate a distributional method for oriented and non-oriented integration over implicit surfaces. This method follows from the link between differential forms and the Dirac distribution concentrated on manifolds. This approach will be applied to compute integrals over real Stiefel manifolds and to obtain a distributional Cauchy theorem for the tangential Dirac operator.

Quaternionic splines, wavelets and prolates

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We investigate the use of Clifford analysis in the construction of

- quaternionic splines on the line
- quaternionic wavelets on the plane, and
- bandpass prolate spheroidal wave functions in arbitrary dimensions.

The first topic generalises constructions of fractional and complex splines and the work has been done jointly with Peter Massopust (Munich). The second topic is joint work with David Franklin and Andrew Morris (Newcastle) and involves the Clifford-Fourier transform and modern techniques of optimisation, especially the Douglas-Rachford algorithm, to construct quaternion-valued, orthogonal, smooth, compactly supported wavelets. The third topic is joint wok with Joe Lakey (Las Cruces) and involves the analysis of Clifford-based generalised translation operators acting on bandlimited functions $f : \mathbb{R}^n \mapsto \mathbb{R}_n$ (where $\mathbb{R}_n$ is the natural Clifford algebra associated with $\mathbb{R}^n$).

Time-Frequency Analysis in the Unit Ball

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As an appropriate analogue of the Euclidean short-time Fourier transform we study a windowed version of the Helgason-Fourier transform on a hyperbolic manifold: the complex unit ball. This voice transform is constructed by means of translations (composition with special Moebius transforms) and modulations (multiplication with the kernel of the Helgason-Fourier transform) of a window function. In absence of a related group representation, the algebraic tools available in the traditional setting have to be replaced in this context by regularity and decay conditions imposed on the fixed window. Under such assumptions, the voice transform can be defined for distributions and displays properties similar to those in the group setting. Modulation spaces (or more general, coorbit spaces) are defined in terms of weighted Lebesgue spaces, and, using partitions of unity, atomic decompositions and Banach frames are obtained.

On the hypercomplex Airy function in the context of quantum Yang-Mills theory

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I outline the quantum pure Yang-Mills theory which is obtained by means of the precanonical quantization

based on the De Donder-Weyl (DW) Hamiltonian theory. It leads to a Dirac-like equation on the Clifford-algebra-valued wave function with the mass term replaced by a the Clifford-algerba-valued DW Hamiltonian operator which controls the mass spectrum of the theory. We show that the eigenvalue problem for the DW Hamiltonian operator can be exactly solved in terms of a certain hypercomplex generalization of the Airy function expressed as the contour integral. We discuss some properties of the solution and the estimation of the gap in the spectrum of the DW Hamiltonian operator.

Spectral Decomposition of Clifford algebras

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In this talk we find a Peirce decompositions of a complex Clifford algebra Cl_n with respect to the complete set of an idempotents. We also show that the above spectral decomposition holds when Cl_n is nondegenerate.

Applications of slice-holomorphic functions to automorphic forms and Bergman kernels

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In this talk we describe the symmetry behavior of slice-holomorphic functions. In particular, we determine the maximal invariance group taking slice-holomorphic functions into themselves. Concretely, we discuss which kinds of Möbius transformations exactly map slice-holomorphic functions onto other slice-holomorphic functions. We prove a conformal covariance property of the Cauchy integral formula. We discuss the problem of isometry between related functions spaces. Furthermore, we explain the explicit construction of the Bergman kernel for slice-holomorphic functions on the unit ball, the half unit ball and the quater unit ball. We round off our talk by giving explicit constructions of automorphic forms in the slice holomorphic setting and explain their relations to Clifford-holomorphic automorphic forms and classical holomorphic forms in several complex variables. This is joint work with F. Colombo, K. Diki and I. Sabadini from the Depatment of Mathematics of the Politecnico di Milano,

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Nonstable quaternionic and orthogonal complexes for k Dirac operators

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We study the operator $D_k = (\partial_1, \ldots, \partial_k)$, sometimes called "$k$ Dirac operators" or "Dirac in k variables". It acts on functions defined on the product $\mathbb{R}^m \times \ldots \times \mathbb{R}^m$ of k copies of the Euclidean space with values in the basic spinor module S_A of the (complexified) Clifford algebra $\mathbb{C}_m$.
The question whether there is an analogue of the Dolbeault complex (called BGG-complex) for the operator D_k, has been studied for many years. The answer is known in several cases and it is well known that there also second order operators appear in the complex.
The BGG-complex for D_k in dimension 4 is an example of so-called nonstable case and it can exist in two versions, according to the choice of a symmetry group. This holds only in the dimension 4, where we can choose either quaternionic or orthogonal symmetry.
We compare both versions of BGG-complexes and show that the orthogonal complex, obtained by Penrose transform, is a refinement of the Baston quaternionic complex, obtained by branching rules.

Fischer decomposition in non-stable cases

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For spinor valued polynomials in k vector variables of dimension m, the Fischer decomposition has been recently proved in the stable range $m \geq 2k$, see arXiv:1708.01426. This decomposition was conjectured by F. Colombo, F. Sommen, I. Sabadini and D. Struppa in 2004. In this case, the symmetry is given by the orthogonal group $O(m)$ and uniqueness of the Fischer decomposition is equivalent to irreducibility of generalized Verma modules for the Howe dual partner $osp(1|2k)$ generated by spherical monogenics. Here spherical monogenics are polynomial solutions of the Dirac equation in k vector variables. In this talk, we shall deal with two possible extensions of this result. First we shall discuss non-stable cases and second a generalization for polynomials in k vector variables on superspaces. In both cases, structure of generalized Verma modules for $osp(1|2k)$ plays an important role.
The talk is based on results obtained jointly with V. Souček.

Application of the discrete potential theory to problems of mathematical physics

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Discrete potential theory is a natural extension of the continuous theory to functions defined on lattices. The idea of the discrete potential theory is to transfer all important aspects of the continuous theory directly to the discrete level. Direct formulation on the discrete level is advantageous for applications in mathematical physics, since important physical quantities, such as for example conservation laws or asymptotic conditions, are modelled and satisfied on the discrete level and not only approximated as in conventional approaches. Thus, in this contribution, we present recent developments in the discrete potential theory on rectangular lattices and its application to planar boundary value problems of mathematical physics in bounded domains.

On application of script geometry to finite element exterior calculus

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Finite element method is probably the most popular numerical method used in different fields of applications nowadays. While approximation properties of the classical finite element method, as well as its various modifications, are well understood, stability of the method is still a crucial problem in practice. Therefore, alternative approaches based not on an approximation of continuous differential equations, but working directly with discrete structures associated with these equations, have gained an increasing interest in recent years. Finite element exterior calculus is one of such approaches. The finite element exterior calculus utilises tools of algebraic topology, such as de Rham cohomology and Hodge theory, to address the stability of the continuous problem. By its construction, the finite element exterior calculus is limited to triangulation based on simplicial complexes. However, practical applications often require triangulations containing elements of more general shapes. Therefore, it is necessary to extend the finite element exterior calculus to overcome the restriction to simplicial complexes. In this paper, the script geometry, a recently introduced new kind of discrete geometry and calculus, is used as a basis for the further extension of the finite element exterior calculus.

Some Properties and Application of Teodorescu Operators Associated with the Helmholtz Equation and the Time-harmonic Maxwell Equations

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In this paper, firstly, we discuss some properties of Teodorescu operator $T_{\psi,\alpha}$ related to the Helmholtz equation and give the integral representation of the solution to the Riemann boundary value problem related to the Helmholtz equation. Then, by using of the previous conclusions, we show some properties of the Teodorescu operator associated with the $\mathcal{N}$ matrix operator and give the integral representation of the solution to the Riemann boundary value problem associated with the $\mathcal{N}$ matrix operator. Finally, by using the relationship between the corresponding Cauchy type integral of the $\mathcal{N}$ matrix operator and the corresponding Cauchy type integral of the Time-harmonic Maxwell equations, we give the integral representation of the solution to the Riemann boundary value problem for homogeneous partial differential equations related to the Time-harmonic Maxwell equations.

On the structure of the singularities of bicomplex holomorphic functions

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The paper Luna-Elizarrarás, Perez-Regalado, Shapiro "On the Laurent Series for bicomplex holomorphic functions", Complex Variables and Elliptic Equations, Vol 62, Issue 9, 2017, 1266-1286, gives a detailed description of the structure and the main properties of the Laurent series in bicomplex analysis. The aim of this talk is to apply the ideas and the results of it for developing the theory of singular points for bicomplex holomorphic functions. A classification of such points is given; the behaviour of bicomplex holomorphic functions near such points is described; the peculiarities of this situation are singled out. The talk is based on is a joint work with C.O. Perez-Regalado and M. Shapiro

Clifford-Valued Cone and Hex Splines

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We introduce an extension of cone splines and box splines hypercomplex orders. These new families of multivariate splines are defined in the Fourier domain along certain s-directional meshes and include as special cases 3-directional box splines and hex splines found in the literature. These cone and hex splines of hypercomplex order generalize univariate complex and hypercomplex B-splines. Explicit time domain representations are derived for these splines on 3-directional meshes. We present some properties of these two multivariate spline families, such as recurrence, decay and refinement. Finally, we show that a bivariate hex spline and its integer lattice translates form a Riesz basis of its linear span.

Radon-Type transforms for holomorphic functions in the Lie ball

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In this talk we will discuss certain Radon-type transforms and their reproducing kernels which are studied in the paper by Sabadini and Sommen. Extending their work we will see some new results I found in joint work with F. Sommen.

Immersions of Riemann surfaces

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We discuss the theory of immersions of Riemann surfaces into $\mathbb{R}^3$ as developed by Kamberov, Norman, Pedit and Pinkall. Their use of the imaginary quaternions is elegant and adds to the theory. We discuss extending the theory to Klein surfaces. This requires including an anti-holomorphic involution of the surface associated with an almost dianalytic structure. We also consider

the use of the imaginary split quaternions which span the Minkowski space $\mathbb{R}^{1,2}$.

A group classification of linear fractional partial differential equation with Laplacian on a plane

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In this talk we consider fractional deformations of second order partial differential operators defined on the plane (such as Laplace and Weinstein-Leutwiler equations). We study basic concepts of local Lie symmetries for operators in the Riemann-Liouville case. Then we study good chosen examples to give classifications for operators. This lead us a canonical way to find proper fractional generalization for PDE's.
This is a joint work with Dr. Aleksej Kasatkin (Ufa) and Dr. Nelson Vieira (Aveiro).

An Almansi type decomposition for slice-regular functions on Clifford algebras

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The aim of the talk is to present an Almansi type decomposition for polynomials with Clifford coefficients, and more generally for slice-regular functions on Clifford algebras. The classical result by Emilio Almansi, published in 1899, dealt with polyharmonic functions, the elements of the kernel of an iterated Laplacian. Here we consider polynomials of the form $P(x) = \sum_{k=0}^{d} x^k a_k$, with Clifford coefficients $a_k \in \mathbb{R}_{0,n}$, an get an analogous decomposition related to zonal polyharmonics. We show the relation between such decomposition and the Dirac (or Cauchy-Riemann) operator and extend the results to slice-regular functions.

Slice Analysis of Several Variables

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Slice analysis of several variables involves a canonical extension of the theory of several complex variables to the non-commutative and non- associative realm via slice technique. The classical theory is a canonical lift of the holomorphic theory from one complex variable to one quaternionic variable and it impels the development of quaternionic Hilbert spaces. In particular, it gives arise the study of spherical-spectrum and has demonstrated potential applications in quantum physics since the spherical-spectrum of a self-adjoint operator for quaternions turns out to be real in contrast to the spectrum non-real. Instead of quaternions, the theory has been successfully extended to octonions, Clifford algebras, and even any alternative algebra but only restricted to the case of one variable. In this talk we extend the theory of several complex variables through the slice technique to several octonionic variables. And we extend the theory of Riemann domains through the slice technique to quaternions and it yields a canonical topology called slice topology in order to be in consistency with the slice structure. We also initiate to study the theory of slice Dirac operator for octonions. The slice Dirac operator is a slice counterpart of the Dirac operator over quaternions. It originates from a new slice structure of octonions with quaternions as a slice and extends the theory of stem functions from abelian setting to nonabelian setting.

Bargmann-Radon transform for axially monogenic functions

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In our work we study a class of monogenic functions called axially monogenic functions. First we present the explicit form for the general Cauchy-Kowalewski extension for axially monogenic functions. Then we determine the Bargmann-Radon transform for these functions, relying on Funk-Hecke theorem in the process.

Multivariable quaternionic functions and power series

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In the talk we consider functions in several quaternionic variables, based on a non-commutative version of Gleason's problem. Our study is based on converging series of monomials in several noncommuting variables. Some questions can be tackled in this framework and we show how to solve the Gleason problem, and an interpolation problem. We also discuss Schur multipliers.
(Joint work with K. Abu-Ghanem, D. Alpay, F. Colombo).

Domains of monogenicity in several quaternionic variables

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The n-Cauchy-Fueter operator plays the role of the Cauchy-Riemann operator in the analysis of several quaternionic variables. It is well known that the Hartog's phenomenon holds in the quaternionic vector space $\mathbb{H}^n$ of dimension n and so it is natural to introduce so called domains of monogenicity in $\mathbb{Q}^n$, these are the domains of holomorphy for the n-Cauchy-Fueter operator.

In the talk I will give sufficient and necessary conditions for a domain in $\mathbb{H}^n$ to be a domain of monogenicity.

Analyticity in the sense of Hausdorff and classes of hyperholomorphic quaternionic functions

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The notion of the Hausdorff analyticity for functions with values in real algebras will be considered together with the Hausdorff derivative for such functions. We analyze what both give for the real quaternions and will compare with the results for several well-known classes of quaternion-valued functions which have the intrinsic notions of the (hyper-) analyticity and the (hyper-) derivative.
The talk is based on a joint work with M.E. Luna-Elizarraras and V. Shpakivskyi.

Two-sided Fourier Transform in Clifford Analysis and its Application

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In this paper, we first define a two-sided Clifford Fourier transform(CFT) and its inverse transformation on L^1 space. Then we study the differential of the two-sided CFT, the k-th power of Fh, Plancherel identity and time-frequency shift of the two-sided CFT.
Finally we discuss the uncertainty principle of the two-sided CFT and give an application of the two-sided CFT to the partial differential equation.

Zeros of slice functions over dual quaternions: theory and applications

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Within the theory of slice functions over alternative $*$-algebras, this talk will focus on the algebra of dual quaternions $\mathbb{DH}$. The zero sets of slice functions, slice regular functions and polynomials over this algebra have peculiar properties. A detailed study of these properties is possible thanks to the characterization of zero divisors within $\mathbb{DH}$.
In addition to its intrinsic interest, this study has useful applications to the open problem of factorizing motion polynomials over dual quaternions. The polynomials in this class, introduced by Hegedüs, Schicho, and Schröcker in 2013, correspond to rational rigid body motions in the Euclidean 3-space. Their factorizations correspond to linkages producing the same motions, so their classification is relevant to mechanism science.
This is a joint work with Graziano Gentili and Tomaso Trinci.

Dirichlet and Bloch Spaces in different Contexts

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What are the diferences and similarities when we consider Bloch and Dirichlet Spaces in different contexts: a) Analytic functions in the unit open complex ball b) Holomorphic functions in the unit ball of C^n c) Monogenic functions in the three dimensional unit ball d) Monogenic functions in the four dimensional unit ball e) Bicomplex holomorphic functions in the bidisk In this talk we present a discussion about that situations

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Ternary Regular Functions

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This talk will present a study of ternary regular functions, as derived from the general theory of hypercomplex analysis. We introduce the notion of regularity, ternary differential operators, and ternary laplacian in this case

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Analysis of the k-Cauchy-Fueter complexes

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The k-Cauchy-Fueter complexes are quaternionic counterparts of $\overline{\partial}$-complex in SCV. We consider the non-homogeneous k-Cauchy-Fueter equations over a domain under the compatibility condition, which naturally leads to a Neumann problem. The method of solving the $\overline{\partial}$-Neumann problem in SCV is applied to this Neumann problem. We introduce notions of k-plurisubharmonic functions and k-pseudoconvex domains, establish the L^2 estimate and solve this Neumann problem over k-pseudoconvex domains in $\mathbb{R}^4$. 0-Cauchy-Fueter complex can be applied to the quaternionic Monge-Ampere operator, and allows us to introduce the notion of a closed positive current in the quaternionic case and to extend several results in complex pluripotential theory to the quaternionic pluripotential theory.

Important Properties of Clifford Möbius Transformation and Its Application

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Important properties of Clifford Möbius transformation and its application are discussed in this paper. First,

some important properties of Clifford Möbius transformation are studied. Then, the application of Clifford Möbius transformation is given, that is, the generalized Jørgensen inequality.

A new Cauchy integral formula in the complex Clifford analysis

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In this talk, we construct an analogue of Bochner-Martinelli Kernel based on theory of functions of several complex variables in complex Clifford analysis, which has generalized complex differential forms with Clifford basis vectors. Using these complex differential forms, we obtain the Stoke's formula of complex Clifford functions which are defined on a domain $\Omega \subset C^{n+1}$ and take values in a complex Clifford algebra $Cl_{1,n}(C)$. Then, we give a Stoke's formula which has a classical form and an analogue of Cauchy-Pompieu formula which is represented by Bochner-Martinelli Kernel, and establish an analogue of Cauchy integral formula in complex Clifford analysis. It is possible to promote these results to complex manifold's corresponding results in the Clifford analysis using the representation by generalized complex differential forms.

S.18 Recent Progress in Evolution Equations

Organisers
Marcello D'Abbicco, Marcelo Rempel Ebert

Scope of the session: The goal of the session "Recent progress in evolution equations" is to discuss the state-of-the-art of qualitative properties of solutions of linear and nonlinear evolution models. We have in mind results for dispersive equations, hyperbolic and p-evolution equations and parabolic equations as well. Among other things well-posedness, asymptotic profile, dissipative estimates, blow-up behavior, critical exponents, influence of low regular coefficients are of interest.

—Abstracts—

Modified different nonlinearities for weakly coupled systems of semilinear effectively damped waves with different time-dependent coefficients in the dissipation terms

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We prove the global existence of small data solution in all space dimension for weakly coupled systems of semi-linear effectively damped wave, with different time-dependent coefficients in the dissipation terms. Moreover, nonlinearity terms $f(t,u)$ and $g(t,v)$ satisfying some properties of the parabolic equation. We study the problem in several classes of regularity.

Decay of the data and regularity of the solution in Schrödinger equations

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We deal with the global Cauchy problem for a Schrödinger type equation

$$\left(D_t - \Delta + \sum_{j=1}^{n} a_j(t,x)D_{x_j} + b(t,x)\right)u(t,x) = 0,$$

$(t,x) \in [0,T] \times \mathbb{R}^n$, under the decay condition $|\Im a_j(t,x)| \leq C/(1+|x|^2)^{\sigma/2}$, $\sigma \in (0,1)$; this condition is known to produce a unique solution with Gevrey regularity of index $s \geq 1$ and loss of an infinite number of derivatives with respect to the data $g(x) = u(0,x)$, for every $s \leq 1/(1-\sigma)$. We consider the case $s > 1/(1-\sigma)$ and we explain how the space where a unique solution exists depends on the decay and regularity of an initial data $g \in H^m$, $m \geq 0$. As a byproduct, we show that if $g \in H^m$ has a decay as $|x| \to \infty$, then the solution has (at least locally) the same regularity as g but a different behavior at infinity.

The influence of oscillations on energy estimates for damped wave models with time-dependent propagation speed and dissipation

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In this talk we discuss the Cauchy problem for damped wave models with time-dependent propagation speed and dissipation. The model of interest is

$$u_{tt} - \lambda^2(t)\omega^2(t)\Delta u + \rho(t)\omega(t)u_t = 0,$$
$$u(0,x) = u_0(x), \;\; u_t(0,x) = u_1(x).$$

The coefficients $\lambda = \lambda(t)$ and $\rho = \rho(t)$ are shape functions and $\omega = \omega(t)$ is a (bounded) oscillating function taking account of C^m properties of the coefficients with a stabilization condition. Moreover, the damping term $\rho(t)\omega(t)u_t$ is assumed to be an "effective like"damping. By introducing these properties simultaneously, more precise analysis is applicable.
Our goal is to derive higher order energy estimates for solutions to the Cauchy problem. We will explain how the interplay between the shape functions and very fast oscillating behavior of the coefficients will influence energy estimates. Finally, our interest is to apply the derived linear estimates to study global existence of small data solution to semi-linear models.
The results of this talk are based on collaborations with Michael Reissig (TU Bergakademie Freiberg).

Regularity of linear partial differential operators with polynomial coefficients

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Let $\mathcal{S}$ be the classical Schwartz space and $\mathcal{S}'$ its dual space of tempered distributions. A linear operator $A : \mathcal{S}' \to \mathcal{S}'$ is said to be globally regular if

$$Au \in \mathcal{S} \;\Rightarrow\; u \in \mathcal{S}, \qquad \forall u \in \mathcal{S}'.$$

Here we study regularity of linear partial differential operators $A(x, D)$ with polynomial coefficients. A well-known sufficient condition for regularity is the condition of global hypoellipticity given by Shubin, but it is far from being necessary and studying regularity for non-global hypoelliptic operators is not trivial, in general. Here we study regularity of linear p.d.o. with polynomial coefficients by using a Wigner type transform and then Cohen classes of the form $Q[w] := \sigma * \mathrm{Wig}[w]$ for a kernel $\sigma \in \mathcal{S}'$. We find classes of regular (but not hypoelliptic) operators, and these classes are quite large because of the freedom in the choice of the kernel σ.

Semilinear p-evolution equations in weighted Sobolev space

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We study the Cauchy problem associated to a class of semilinear p-evolution equations in a suitable scale of weighted Sobolev spaces. Applications concern KdV-type equations and semilinear Schrödinger-type equations.

Existence and asymptotic properties for dissipative semilinear second order evolution equations with fractional Laplacian operators

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In this work we study the existence and uniqueness of solutions for a generalized second order semilinear evolution equation with Laplace operators depending on fractional powers. We also study the decay rates of the solutions and associated energy in the sense of L^2 norm. For the associated linear equation, using an asymptotic expansion of the solution of the problem in the Fourier space we show optimality of the decay rates obtained for certain powers of the involved Laplacian operators . Moreover, we study the case of super damping using an improvement of the standard case and we prove optimality of the decay rates for the L^2 norm. Our results generalize some previous works that deal with particular cases of fractional exponents of the Laplacian operators. Key words: Plate/Boussinesq type equation; Existence and uniqueness; Fractional Laplacian; Decay rates; Asymptotic expansion; Super-damping; Optimal decay rates. Joint with: Juan Torres Espinoza (UFSC-Brazil) and Ryo Ikehata (Hiroshima University.

Weakly coupled systems of semilinear elastic waves with different damping mechanisms in 3D

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We consider the following Cauchy problem for weakly coupled systems of semilinear damped elastic waves with a power source nonlinearity in three dimensions:

$$U_{tt} - a^2\Delta U - \big(b^2 - a^2\big)\nabla \mathrm{div}\, U + (-\Delta)^\theta U_t = F(U), \quad (t,x) \in (0,\infty)\times\mathbb{R}^3,$$

where $U = \big(U^{(1)}, U^{(2)}, U^{(3)}\big)^{\mathrm{T}}$ with $b^2 > a^2 > 0$ and $\theta \in [0,1]$. Our interests are some qualitative properties of solutions to the corresponding linear model with vanishing right-hand sides and the influence of the value of θ on the exponents p_1, p_2, p_3 in the nonlinear term $F(U) = \big(|U^{(3)}|^{p_1}, |U^{(1)}|^{p_2}, |U^{(2)}|^{p_3}\big)^{\mathrm{T}}$ to get results for the global (in time) existence of small data solutions.

The results of this talk are based on collaborations with Michael Reissig (TU Bergakademie Freiberg). [1] Chen W, Reissig M. Weakly coupled systems of semilinear elastic waves with different damping mechanisms. *Math. Methods Appl. Sci.* (accepted for publication)

Asymptotics for a semilinear damped plate equation with time-dependent coefficients

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In this talk, we will discuss the asymptotic profile of the solution to a semilinear damped plate equation with time-dependent coefficients

$$u_{tt} + \Delta^2 u - \lambda(t)\Delta u + u_t = |u|^p, \quad t > 0,\ x \in \mathbb{R}^n$$

assuming small initial data. Here $\lambda(t)$ is a nonincreasing function whose decay speed at infinity regulates if the asymptotic profile is described by a heat equation, or by a fourth order parabolic equation.

L^1 estimates for oscillating integrals and application to parabolic like semi-linear structurally damped σ-evolution models

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In the talk we discuss L^1 estimates for oscillating integrals by applying the theory of modified Bessel functions combined with Faà di Bruno's formula and their applications to the Cauchy problems for semi-linear structurally damped σ-evolution models. The model of interest is

$$u_{tt} + (-\Delta)^\sigma u + \mu(-\Delta)^\delta u_t = f(u, u_t),$$
$$u(0,x) = u_0(x),\ u_t(0,x) = u_1(x)$$

where $\sigma \geq 1$, $\mu > 0$, $\delta \in (0, \frac{\sigma}{2})$ and the function $f(u, u_t)$ stands for the power nonlinearities $|u|^p$ and $|u_t|^p$. The novelty is to prove the global (in time) existence of small data Sobolev solutions to the above semi-linear models from suitable function spaces basing on L^q spaces by using $L^q \cap L^m - L^q$ estimates not necessarily on the conjugate line and some modern tools from Harmonic Analysis. Moreover, we show how the flexibility of parameters m and q brings some benefits to relax the restrictions to the admissible exponents p.

The results of this talk are based on collaborations with Michael Reissig (TU Bergakademie Freiberg).

On regularity of solutions to two-dimensional Zakharov-Kuznetsov equation

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The initial and initial-boundary value problems in different domains are considered for two-dimensional Zakharov-Kuznetsov equation $u_t + bu_x + u_{xxx} + u_{xyy} + uu_x = 0$. Results both on solubility of these problems in regular functional spaces and on internal regularity of weak solutions are established.

On Traveling Solitary Waves for nonlinear Half-Wave Equations

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We consider nonlinear half-wave equations with focusing power-type nonlinearity of order p. with exponents $1 < p < \infty$ for $d = 1$ and $1 < p < (d+1)/(d-1)$ for $d \geq 2$. We study traveling solitary waves of the form

$$u(t, x) = e^{i\omega t} Q_v(x - vt)$$

with frequency $\omega \in \mathbb{R}$, velocity $v \in \mathbb{R}^d$, and some finite-energy profile $Q_v \in H^{1/2}(\mathbb{R}^d)$, $Q_v \not\equiv 0$. We prove that traveling solitary waves for speeds $|v| \geq 1$ do not exist. As a second main result, we show that small data scattering fails to hold for the focusing half-wave equation in any space dimension. The proof is based on the existence and properties of traveling solitary waves for speeds $|v| < 1$. Finally, we discuss the energy-critical case when $p = (d+1)/(d-1)$ in dimensions $d \geq 2$.

Time-dependent propagation speed vs strong damping

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We consider a degenerate abstract wave equation with a time-dependent propagation speed. We investigate the influence of a strong dissipation, namely a friction term that depends on a power of the elastic operator. We discover a threshold effect. If the propagation speed is regular enough, then the damping prevails, and therefore the initial value problem is well-posed in Sobolev spaces. Solutions also exhibit a regularizing effect analogous to parabolic problems. As expected, the stronger is the damping, the lower is the required regularity. On the contrary, if the propagation speed is not regular enough, there are examples where the damping is ineffective, and the dissipative equation behaves as the non-dissipative one.

$L^1 - L^1$ estimates for the strongly damped plate equation

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We will derive L^1 estimates for the solution for the strongly damped plate equation,

$$u_{tt} + \Delta^2 u + \Delta^2 u_t = 0 \quad x \in \mathbb{R}^n,\, t \in \mathbb{R}_+,$$
$$u(0, x) = u_0(x),\ u_t(0, x) = u_1(x).$$

In particular, we will prove that

$$\|u(t, \cdot)\|_{L^1} \leq C\,(1+t)^{\frac{n}{4}} \big(\|u_0\|_{L^1} + (1+t)^{\frac{1}{2}} \|u_1\|_{L^1}\big),$$

for any $t \geq 0$, in space dimension $n \geq 5$.

Infinite dimensional Duffing-like evolution equations

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We consider an abstract evolution equation inspired by some physical models for damped oscillations of a beam subject to external loads or magnetic fields, and shaken by a transversal force. This leads to an equation of order two with linear damping, a linear term (the elastic restoring force), a nonlinear term of Duffing type (due to the external load or the magnetic fields), and a time dependent forcing term (the transversal force). The Duffing form of the nonlinear term implies that, when there is no external force, the system has three stationary positions, two stable and one unstable, and all solutions are asymptotic for large times to one of these stationary solutions. We show that this pattern extends to the case where the external force is bounded and small enough, in the sense that all solutions remain close, as time goes to infinity, to one of the three stationary solutions to the unforced equation, within a distance depending on the size of the forcing term. Moreover, two solutions that are eventually close to the same stationary point are actually asymptotic to each other, and therefore in this regime solutions have only three possible asymptotic profiles. One interesting feature if that boundary conditions have an important influence on the functional analytic setting. The case of hinged endpoints is easier to deal with because the operator involved in the elastic part commutes with the operator in the Duffing term. More difficult is the case of clamped endpoints, because now the operators do not commute, and new functional tools are needed in the analysis.

Some properties of time dependent propagation speed of the wave equation for the estimates of low frequency energy

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We consider the energy estimate of the solution to the Cauchy problem of the wave equation with time dependent propagation speed:

$$\begin{cases} \partial_t^2 u - a(t)^2 \Delta u = 0, \ \ (t,x) \in (0,T] \times \mathbb{R}^n, \\ u(0,x) = u_0(x), \ \ \partial_t u(0,x) = u_1(x), \ \ x \in \mathbb{R}^n, \end{cases}$$

where $a(t)$ is not Lipschitz continuous at $t = T$ hence it may occur some loss of regularity as $t \to T - 0$. The main purpose of my talk is to introduce a new property of $a(t)$ which is effective in particular for the estimate to of the low frequency energy.

On a criterion for log-convex decay in non- selfadjoint dynamical systems

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Some recent results on the well-posedness of a large class of variational evolution equations are first recalled as a motivation. Then the short-time and global behaviour are studied in an autonomous case defined by a non-self-adjoint generator inducing a uniformly bounded holomorphic semigroup in a Hilbert space. A general necessary and sufficient condition is introduced under which the norm of the solution is shown to be a log-convex and strictly decreasing function of time, and differentiable also at the initial time with a derivative controlled by the lower bound of the generator. The recently obtained injectivity of holomorphic semigroups plays a pervasive role in the arguments.

Blow-up for Quasi-Linear Wave Equations with time dependent potential

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Many papers deal with nonlinear wave equations with low order terms, such as damping terms. The main question concerns the influence of the low order terms on the critical exponents of global existence or blow-up or scattering. With suitable transformations these equations can be studied as nonlinear wave equations with a time dependent potential in the nonlinear terms. In a joint project with Vladimir Georgiev we are working on quasi-linear wave equations having particular structure. In this talk we present a blow-up result in three-dimensional case with polynomial potential. An interaction between the decay of the potential and the structure of the quasilinear term appears. Coming back to nonlinear wave equation with scale invariant damping, we can add some missing tiles on the analysis of such problem.

Sigma-evolution models with low regular time-dependent structural damping: effective and non-effective dissipation

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In this work, we consider $\sigma-$evolution models under effects of a damping term represented by the action of a fractional Laplacian operator and a time-dependent coefficient $b(t)(-\Delta)^{\theta} u_t$. The objective of this work is to obtain $L^p - L^q$ decay rates, with $1 \leq p \leq 2 \leq q \leq \infty$, for the solution and its first derivative in time, considering low regularity in the coefficient $b = b(t)$. More precisely, considering a suitable t_0, we take $b(t)$ "confined" in the curve $g(t) := (1+t)^{\alpha} \ln^{\gamma}(1+t)$ for $t \geq t_0$. In this context, when compared with previous results that assume more regularity in the function b, we obtain the same decay rates for the solution when $\gamma = 0$ and we reach better rates for $\gamma \neq 0$. For the first derivative in time of the solution, we derive improved rates even for the case $\gamma = 0$. Written in a equivalent manner, an important conjecture was asserting: "For $\theta = 0$, when the coefficient of the damping is effective, without further assumptions on derivatives of the coefficient is still possible to achieve the same known decay rates for the problem". In the present work we provide an answer to the conjecture, showing, in addition, that there are other situations in which the conjecture remains valid, for example $\theta \neq 0$ or even in the case that the damping is non-effective.

Critical exponent for the semilinear damped wave equation with polynomial decaying propagation speed

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We consider the Cauchy problem on $\mathbf{R_0^+} \times \mathbf{R^n}$ for the semilinear damped wave equation

$$u_{tt}(t,x) - a^2(t)\Delta u(t,x) + b(t)u_t(t,x) = |u|^p$$

with decreasing in time coefficients, the propagation speed $a(t) = (1+t)^{-\ell}$, $0 < \ell < 1$, the scale-invariant dissipation $b(t) = \beta(1+t)^{-1}$, $\beta > 0$, and a nonlinear power term of order $p > 1$. The solution u^0 of the corresponding linear problem will be represented in the explicit form using Fourier multipliers operators with multipliers expressed in terms of special functions. Our main goal is to prove the global existence of u in time when initial data belongs to the spaces $H^1(\mathbf{R^n})$ and $L^2(\mathbf{R^n})$. We are focused in finding the critical exponent $p_c(n)$ such that if $1p_c(n)$ the global solution has the same long time behavior of u^0. In order to estimate u we use Duhamel's principle to represent u and then we apply $L^2 - L^2$ estimates of u^0.

Global solutions for the semilinear diffusion equation in the de Sitter spacetime

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The Cauchy problem for the semilinear diffusion equation is considered in the de Sitter spacetime with the spatial zero-curvature. Global solutions and their asymptotic behaviors for small initial data are obtained for positive and negative Hubble constants. The effects of the spatial expansion and contraction are studied on the problem.

Some estimates of solutions of perturbed Helmholtz equations

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In this talk, we consider the stationary problem of wave equations with dissipation in an exterior domain $\Omega \subset \mathbb{R}^N$ $(N \geqq 3)$. For the estimate of solutions of this problem, the existing results require an assumption of the smallness of the coefficient function of dissipation. To overcome this difficulty, we introduce new inequalities needed to estimate the solution.

Recent progress in semilinear wave equations in the scale-invariant case

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In this talk, we provide an overview of semilinear wave models with scale-invariant damping and mass with different nonlinear terms. In particular, we shall emphasize how the critical conditions for the exponents in the nonlinear terms turn out to be shifts of the critical exponents (respectively, critical curves in the case of weakly coupled systems) for classical wave and damped wave models. This phenomenon is strictly related to the threshold nature of the time-dependent coefficients for the damping and mass terms in the scale-invariant case.
This talk is based on joint works with Michael Reissig (TU Bergakademie Freiberg), Wenhui Chen (TU Bergakademie Freiberg), Wanderley Nunes do Nascimento (Federal University of Rio Grande do Sul) and Ziheng Tu (Zhejiang University).

Some recent results on periodic waver waves

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We will consider the full water waves problem, modeling the motion of waves like those on the surface of the deep ocean, and discuss the question of long-time stability for small solutions. While several results have been proven in recent years for the problem set on the whole Euclidean space, the motion of periodic waves is less understood. I will present two recent results in this direction: a result on the long-time stability of 1d periodic interfaces under the influence of gravitational forces which settles a conjecture by Zakharov and Dyachenko on the almost integrability of the equations (joint work with M. Berti and R. Feola) and a result on the full 3d periodic gravity-capillary problem (joint work with A. Ionescu).

L^p-L^q estimates for the damped wave equation and the critical exponent for the nonlinear problem with slowly decaying data

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We study the Cauchy problem of the damped wave equation

$$\partial_t^2 u - \Delta u + \partial_t u = 0$$

and give sharp L^p-L^q estimates of the solution for $1 \leq q \leq p < \infty$ $(p \neq 1)$ with derivative loss. This is an improvement of the so-called Matsumura estimates. Moreover, as its application, we consider the nonlinear problem with initial data in $(H^s \cap H_r^\beta) \times (H^{s-1} \cap L^r)$ with $r \in (1, 2]$, $s \geq 0$, and $\beta = (n-1)|\frac{1}{2} - \frac{1}{r}|$, and prove the local and global existence of solutions. In particular, we prove the existence of the global solution with small initial data for the critical nonlinearity with the power $1 + \frac{2r}{n}$.

S.19 Spectral Theory of Partial Differential Equations

Organisers
James Kennedy, Davide Buoso

Scope of the session: This session seeks to bring together researchers working on spectral-theoretic aspects of partial differential equations, including but not limited to those coming from mathematical physics. This covers both the analysis of eigenvalues of partial differential operators with discrete spectrum, as well as properties of operators with essential spectrum, and nonlocal and non-self-adjoint problems. The emphasis is anticipated to be on topics such as perturbation theory and spectral dependence on parameters, weights etc., non-convergence results and "term-coming-from-nowhere" effects, spectral asymptotics, location of the spectrum and isolated eigenvalues, variational methods, and spectral geometry and shape optimization, but submissions are welcome from all related areas.

—Abstracts—

Some sharp spectral inequalities

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We prove upper and lower bounds for Dirichlet eigenvalues of linear and nonlinear operators.

Semiclassical bounds for spectra of biharmonic operators

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We consider Riesz means of the eigenvalues of the biharmonic operator subject to various boundary conditions, and we improve the known sharp semiclassical bounds in terms of the volume of the domain with a second term with the expected power. We obtain such estimates by means of the averaged variational principle. This method intrinsically also yields two-sided bounds for individual eigenvalues, which are semiclassically sharp. Based on a joint work with L. Provenzano and J. Stubbe.

Spectral theory for beams with intermediate piers and related nonlinear evolution problems

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We prove a complete spectral theorem for the linear eigenvalue problem for beams with intermediate piers, explicitly determining the expression of the eigenfunctions and discussing in detail their nodal properties. This provides a suitable tool in order to study the qualitative properties of the solutions of different fourth-order nonlinear evolution problems (possibly with nonlocal effects) motivated by the dynamics of suspension bridges. In this framework, we discuss the optimal placement of the piers in terms of suitable notions of stability. Joint work with Filippo Gazzola (Dipartimento di Matematica, Politecnico di Milano).

On the structure of the spectrum of regular boundary value problems for differential equations

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So far, an example of a regular boundary value problem for differential equations, whose spectrum (except for the empty set) is a finite set, has not been found. In the present work, using the method of anti-apriori test function estimates, it was possible to establish that the spectrum of differential operators (with constant coefficients in a certain subdomain) generated by regular boundary conditions is either empty or infinite. In the case of hypoellipticity of the considered differential operator with variable coefficients everywhere, the absence of its finite spectrum is also established.

On homogenization of periodic parabolic systems

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The talk is devoted to homogenization of periodic differential operators. In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider matrix elliptic second order differential operator B_ε. It is assumed that B_ε is positive definite. Coefficients of the operator are periodic with respect to some lattice in $\mathbb{R}^d$ and depend on $\mathbf{x}/\varepsilon$. For the semigroup $e^{-B_\varepsilon t}$, $t>0$, we obtain approximation in the L_2-operator norm with the error estimates of the order $O(\varepsilon)$ and $O(\varepsilon^2)$ for fixed time:

$$\|e^{-B_\varepsilon t}-e^{-B^0 t}\|_{L_2\to L_2}\leqslant C_1\varepsilon(t+\varepsilon^2)^{-1/2}e^{-C_2 t},$$
$$\|e^{-B_\varepsilon t}-e^{-B^0 t}-\varepsilon K(\varepsilon;t)\|_{L_2\to L_2}\leqslant C_3\varepsilon^2 t^{-1}e^{-C_2 t}.$$

Here B^0 is the effective operator with constant coefficients and $K(\varepsilon;t)$ is the corrector. We apply spectral approach to homogenization problems developed by M. Birman and T. Suslina. The method of investigation is based on the scaling transformation, the Floquet-Bloch theory, and the analytic perturbation theory.

Eigenvalues of the Third Boundary Value Problem for the Heat Equation

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The talk will present some challenges and insights in dealing with the eigenvalue problem of the third boundary condition for the heat equation on a line segment. It is shown that for the full third boundary condition, unlike the mixed boundary condition, in general one cannot have unique solution. One could have one or two solutions or no solution at all. The most significant challenge occurs in determining the number of solutions , i.e., how to spot the condition which separates the one solution case from the two solution case. The problem is studied utilizing separation of variables method. In all cases, solvability conditions and explicit solutions are provided.

On the negative spectrum of generalized Hamiltonians defined on metric graphs

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On a connected compact metric graph, we show that the number of negative eigenvalues, of a generalized Hamiltonian, is bounded (up to a multiplicative constant) by an integral of the potential.

Coercive estimates for a second-order differential equation with oscillating drift

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We study the following differential equation:

$$-y'' + r(x)\,y' + q(x)\,y = f(x), \qquad (1)$$

where $x \in R = (-\infty, +\infty)$ and $f \in L_p(R)$, $1 < p < +\infty$. We assume that r, q are, respectively, continuously differentiable and continuous functions. The purpose of this work is to find some conditions for the coefficients r and q such that for any $f \in L_p(R)$ there exists a unique solution y of the equation (1) and the following estimate holds:

$$\|y''\|_p + \|ry'\|_p + \|qy\|_p \le C\,\|Ly\|_p, \qquad (2)$$

where $\|\cdot\|_p$ is the norm in $L_p(R)$.
The equation (1) and its multidimensional generalizations with unbounded coefficients have used in stochastic analysis, biology and financial mathematics. For this reason, interest in these equations has considerably grown in recent years. A number of researches were devoted to the case that the coefficient r are controlled by q. Without the dominating potential q, the case that r growth at most as $|x|\ln(1+|x|)$ were considered by Lunardi A. and Vespri V. (1997), Metafune G. (2001), Prüss J., Rhandi A. and Schnaubelt R. (2006), Hieber M. and others (2009).
In the present work, we study the equation (1) in assumpsion that the coefficient r can quickly grow and fluctuate, and it does not depend on q.

Quantitative aspects for homogenisation problems in $\mathbb{R}^n$

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In this talk we provide a methodology to obtain optimal in order operator norm estimates for homogenisation problems for PDE-systems in $\mathbb{R}^n$. Assuming that the operators in question admit a so-called fibre decomposition, we use elementary Hilbert space methods to compare PDEs with highly oscillatory periodic coefficients with a suitable homogenised PDE. The results are applicable to elliptic systems or static Maxwell's equations. With these results, estimates for time-dependent problems can be derived yielding quantitative homogenisation error estimates for both parabolic and hyperbolic problems.
The talk is based on joint work with Shane Cooper (Durham). Most of the results can be found in [Cooper, S. and Waurick, M. Fibre Homogenisation; Journal of Functional Analysis 276(11):3363–3405, 2019].

Spectral stability for the Maxwell equations in a cavity

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Given a simply connected Lipschitz domain Ω in $\mathbb{R}^3$, we consider the following eigenvalue problem arising in the theory of electromagnetism

$$\begin{cases} \operatorname{curl}\operatorname{curl} v = \lambda v, & \text{on } \Phi(\Omega), \\ \operatorname{div} v = 0, & \text{on } \Phi(\Omega), \\ v \times \nu = 0, & \text{on } \partial\Phi(\Omega), \end{cases}$$

in the unknown vector fields v (the eigenvectors) and $\lambda \in \mathbb{R}$ (the eigenvalues). Here Φ is taken in a suitable class of diffeomorphism in $\mathbb{R}^3$ and ν denotes the unit outer normal to $\partial\Phi(\Omega)$. We discuss the stability of eigenvalues and eigenvectors upon the perturbation of Φ. In particular, we prove the analytic dependence of the symmetric functions of the eigenvalues, and we provide Hadamard-type formulas for the corresponding differentials.
Based on a joint work with Pier Domenico Lamberti.

Conditions of maximal regularity for a third-order differential equation with fast growing intermediate coefficients

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In this report we consider the third-order differential equation with variable coefficients of the form

$$Ly := -y''' + r(x)y'' + q(x)y' + s(x)y = f(x), \qquad (1)$$

where $x \in R = (-\infty, +\infty)$, $f \in L_2(R)$, and r , q and s are smooth functions. We discuss the correct solvability (1) and the possibility of performing for its solution y of the following estimate

$$\|-y'''\|_2 + \|ry''\|_2 + \|qy'\|_2 + \|sy\|_2 \le C\|f\|_2. \qquad (2)$$

If (2) is fulfilled, then the solution y of the equation (1) is called the maximally L_2 -regular, and the inequality (2) is called the of maximal L_2 -regularity estimate.
Maximal regularity is an important tool in the theory of linear and nonlinear differential equations. The question of the maximal regularity for the second order differential equations was investigated by P.C. Kunstmann, W. Arendt, M. Duelli, G. Metafune, D. Pallara, J. Prüss, R. Schnaubelt, A. Rhandi and etc.
The problems of the solid state physics, atmospherical physics, electrohydrodynamics, as well as the processes flowing in environments, that able to remember lead to the third-order differential equations (1). The case of bounded domains and smooth coefficients are well understood and sufficiently well described in the known literature. In the singular case, although the solution of

the odd-order equation (1) is smooth, but it may not belong to any Sobolev space, or its belongs to the Sobolev space is not known in advance. These facts causes some difficulties for study of (1).
In the case that $r = q = 0$ the maximal regularity of the solution of (1) is previously investigated by M.B. Muratbekov, M. Otelbaev, A. Birgebaev and etc. Their results are applicable to the case (1) that r and q are controlled by s. In the present paper, we investigate the case, that r and q in (1) do not depend on s.

S.20 Theory and Applications of Boundary-domain Integral and Pseudodifferential Operators

Organisers
Sergey Mikhailov, David Natroshvili

Scope of the session: The goal of this session is to discuss recent progress in the theory of boundary-domain integral and pseudodifferential operators and their applications in Mathematical Physics, Solid and Fluid Mechanics, Wave Scattering Problems, Engineering Mathematics, etc.

—Abstracts—

Two-operator BDIEs for Variable-Coefficient Neumann Problem with General Data

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The Neumann problem for the second-order scalar elliptic differential equation with variable coefficient and with the PDE right-hand side from $\widetilde{H}^{-1}(\Omega)$ is considered. Applying the two-operator approach, the problem is reduced to two different systems of two-operator boundary-domain integral equations (BDIEs). It is proved that both BDIE systems are equivalent to the original BVP. Invertibility of Boundary-Domain Integral operators is investigated.

Boundary-Domain Integral Equation systems to the Mixed BVP for Compressible Stokes Equations with Variable Viscosity in 2D

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The Boundary-Domain Integral Equations (BDIEs) for the mixed boundary value problem for a compressible Stokes system of partial differential equations with variable coefficient in 2D is considered . An appropriate parametrix (Levi function) is used to reduce this boundary value problem (BVP) to the BDIE systems. Although the theory of BDIEs in 3D is well developed, the BDIEs in 2D need a special consideration due to their different equivalence properties. As a result, we need to set conditions on the domain or on the corresponding function spaces to ensure the invertibility of parametrix-based integral layer potentials and hence the unique solvability of BDIE systems. Equivalence of the BDIE systems to the mixed BVP and invertibility of the matrix operators associated with the BDIE systems are proved.
(Based on a joint work with Mulugeta A. Dagnaw and Sergey E. Mikhailov)

Uniform Convergence of Double Vilenkin-Fourier Series

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My talk is about the uniform convergence problem for the rectangular partial sums of double Fourier series on bounded Vilenkin group of the functions of partial bounded oscillation.

Two-operator Boundary-Domain Integral Equations for variable coefficient Neumann boundary value problem in 2D

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The Neumann boundary value problem (BVP) for the second order "stationary heat transfer" elliptic partial differential equation with variable coefficient is considered in two-dimensional bounded domain. Using an appropriate parametrix (Levi function) and applying the two-operator approach, this problem is reduced to some systems of boundary-domain integral equations (BDIEs). The two-operator BDIEs in 2D have special consideration due to their different equivalence properties as compared to higher dimensional case due to the logarithmic term in the parametrix for the associated partial differential equation. Consequently, we need to set conditions on the domain or function spaces to insure the invertibility of the corresponding layer potentials, and hence the unique solvability of BDIEs. Equivalence of the two operator BDIE systems to the original Neumann BVP, BDIEs solvability, uniqueness/non uniqueness of the solution, as well as Fredholm property and invertibility of the BDIE operator are analysed. Moreover, the two operator boundary domain integral operators for the Neumann BVP are not invertible, and appropriate finite-dimensional perturbations are constructed leading to invertibility of the perturbed operators.

Dynamical interaction problems of acoustic waves and thermo-electro-magneto elastic structures

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We investigate the dynamical transmission problems arising in the model of fluid-solid acoustic interaction

when thermo-electro-magneto elastic body is embedded in an unbounded fluid domain. The physical kinematic and dynamic relations mathematically are described by appropriate boundary and transmission conditions. We investigate the uniqueness of solutions to the dynamical transmission problems and analyze the corresponding transmission pseudo-oscillation problems, which are obtained from the dynamical problems by the Laplace transform.
The solvability of the transmission pseudo-oscillation problems are analyzed by the potential method and theory of pseudodifferential equations in appropriate Sobolev-Slobodetskii spaces. Further using the inverse Laplace transform we obtain solvability of the dynamical transmission problems. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant number FS-18-126).

Approximate Solution for Thin Plates in an Infinite Domain

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Many times, solutions of boundary value problems for a mathematical model cannot be computed explicitly, but can be approximated to within acceptable tolerances by means of expansions in a complete set of functions in a Hilbert space such as L^2. The usefulness of the method increases when the choice of these functions is based on the structure of the layer potentials generated by the problem. This type of expansion may encounter difficulties in the case of an infinite domain, where the solution must fit a certain prescribed far-field pattern. Often, such a pattern requires the a priori knowledge of what is known as a rigid displacement in elasticity theory, which is not normally readily available. In this paper, we address the question of numerical solvability for the Dirichlet problem associated with the bending of a thin elastic plate with transverse shear deformation that occupies the complement to the plane of a finite domain bounded by a closed curve. The generalized Fourier series procedure constructed for this model is illustrated by means of a computational example where the domain is the outside of a circle.
Joint work with Dale Doty, dale-doty@utulsa.edu

Boundary-Domain Integral Equation Systems to the Dirichlet BVP for Stokes Equations with variable viscosity in 2D

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The Dirichlet boundary value problem for the steady-state Stokes system of partial differential equations for a compressible viscous fluid with variable viscosity coefficient is considered in two- dimensional bounded domain. Using an appropriate parametrix, this problem is reduced to a system of direct segregated Boundary-Domain Integral Equations (BDIEs). The BDIEs in the two-dimensional case have special properties in comparison with the three dimension because of the logarithmic term in the parametrix for the associated partial differential equation. Consequently, we need to set conditions on the function spaces or on the domain for the invertibility of corresponding parametrix-based hydrodaynamic single layer potential and hence the unique solvability of BDIEs. Equivalence of the BDIE systems to the Dirichlet BVP and invertibility of the corresponding boundary-domain integral operators are shown.

Boundary-Domain Integral Equation Systems to the Neumann BVP for a compressible Stokes System with variable viscosity in 2D

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The Neumann boundary value problem for the steady-state Stokes system of partial differential equations for a compressible viscous fluid with variable viscosity coefficient is considered in two- dimensional bounded domain. Using an appropriate parametrix, this problem is reduced to a system of direct segregated Boundary-Domain Integral Equations (BDIEs). The BDIEs in the two-dimensional case have special properties in comparison with the three dimension because of the logarithmic term in the parametrix for the associated partial differential equation. Consequently, setting conditions on the function spaces ensures the invertibility of corresponding parametrix-based layer potentials and hence the unique solvability of BDIEs. Equivalence of the BDIE systems to the Neumann BVP and invertibility of the corresponding boundary-domain integral operators are shown.

Mixed Boundary Value Problems for an Elliptic Equation in a Lipschitz Domain

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The mixed boundary value problems for an elliptic equation in a planar Lipschitz domain $\Omega \subset \mathbb{R}^2$ is investigated. The BVP is considered in a non-classical setting when a solution is sought in the Bessel potential spaces $\mathbb{H}^s_p(\Omega)$, $s > 1/p$, $1 < p < \infty$. By applying a localization the BVP is reduced to the investigation of the same problems at model angular domains, as many as the angular points on the boundary of the domain.
The model problems in an angular domain are investigated using the potential method by reducing them to an equivalent boundary integral equation (BIE) in the Sobolev-Slobodečkii space on a semi-infinite axes $\mathbb{W}^{s-1/p}_p(\mathbb{R}^+)$, which is of Mellin convolution type. By applying the recent results on Mellin convolution equations in the Bessel potential spaces obtained by V. Didenko & R. Duduchava in 2016, explicit conditions of the unique solvability of this BIE in the Sobolev-Slobodečkii $\mathbb{W}^r_p(\mathbb{R}^+)$ and Bessel potential $\mathbb{H}^r_p(\mathbb{R}^+)$ spaces for arbitrary r are found and used to write explicit conditions

for the Fredholm property and unique solvability of the initial model BVPs for the Helmholtz equation in the above mentioned non-classical setting.

Boundary Domain Integral Equations for Diffusion Equation in Non-homogeneous Media with Dirichlet Boundary Conditions based on a New Family of Parametrices

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The Dirichlet problem for the diffusion equation in not homogeneous media is reduced to a system of direct segregated parametrix-based Boundary-Domain Integral Equations (BDIEs). We use a parametrix different from the one studied in (Chkadua et al, 2009) . Mapping properties of the potential type integral operators appearing in these equations are presented in appropriate Sobolev spaces. We prove the equivalence between the original BVP and the corresponding BDIE system. The invertibility and Fredholm properties of the boundary-domain integral operators are also analysed. Possible extension of these results to exterior domains is also discussed.

Mellin pseudodifferential operators with non-regular symbols and their applications

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The talk is devoted to applications of Mellin pseudodifferential operators with non-regular symbols to studying the Fredholmness in Banach algebras of singular integral operators with piecewise quasicontinuous coefficients on weighted Lebesgue spaces with Muckenhoupt weights. Applications to Banach algebras of convolution type operators are also considered.

Layer potentials and exterior boundary problems in weighted Sobolev spaces for the Stokes system with L^∞ elliptic coefficient tensor

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We obtain well-posedness results in L^p-based weighted Sobolev spaces for transmission and exterior boundary value problems for the anisotropic Stokes system with L^∞ strongly elliptic coefficient tensor in bounded and complementary exterior Lipschitz domains of $\mathbb{R}^n$, $n \geq 3$. First, we use a variational approach that reduces two linear transmission problems for the anisotropic Stokes system to equivalent mixed variational formulations with data in L_p-based weighted Sobolev and Besov spaces. We show that such a mixed variational formulation is well-posed in the space $\mathcal{H}^1_p(\mathbb{R}^n)^n \times L_p(\mathbb{R}^n)$, $n \geq 3$, for any p in an open interval containing 2. These results are used to define the Newtonian and layer potential operators for the considered anisotropic Stokes system. Various mapping properties of these operators are also obtained. A similar variational approach is used to show the well-posedness of the exterior Dirichlet problem and of the exterior mixed problem for the Stokes system with L^∞ elliptic coefficient tensor in an exterior Lipschitz domain Ω_- of $\mathbb{R}^n$, $n \geq 3$, and in L^p-based weighted Sobolev spaces $\mathcal{H}^{1,p}(\Omega_-)^n \times L^p(\Omega_-)$. For $p = 2$, the solutions of the exterior Dirichlet and Neumann problems are obtained in terms of Newtonian and layer potentials. This talk is based on joint work with Sergey E. Mikhailov (Brunel University London) and Wolfgang L. Wendland (Stuttgart University).

Solution of the Riemann boundary value problem in the case when the free term belongs to the grand variable exponent Lebesgue space $L^{p(t),\theta}(\Gamma)$ when $\min_\Gamma p(t) = 1$

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In our talk we plan to present our results concerning the following topics: i) Singular integrals in grand variable exponent Lebesgue spaces; ii) An extension of notion of Lebesgue integral; iii) Generalized Cauchy type integrals and their properties; iv) Solution of Riemann BVP; v) Boundary singular integral equation of Harmonic Analysis in mixed weighted grand function spaces. We intend to give some application to the approximation theory.
Acknowledgement. The work was supported by Shota Rustaveli National Science Foundation grant (No. DI-18-118).

Singular integrals in weighted grand variable exponent Lebesgue spaces

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The boundedness of maximal and singular integral operators is established in weighted grand variable exponent Lebesgue spaces with power-type weights. The same problem for commutators of singular integrals is also studied. These spaces unify two non-standard classes of function spaces, namely, grand Lebesgue and variable exponent Lebesgue spaces. The spaces and operators are defined, generally speaking, on quasi-metric measure spaces with doubling measure. Exponents of spaces satisfy log-Hölder continuity condition.
Acknowledgement. The work was supported by the Shota Rustaveli National Science Foundation grant (No. DI-18-118).

Analysis of Boundary-Domain Integral Equations for the Variable-Viscosity Compressible Robin-Stokes BVP in L_p-based Spaces

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We consider Boundary-Domain Integral Equations (BDIEs) associated with the Robin boundary value problem for the stationary compressible Stokes system in L_p-based Sobolev spaces in a bounded Lipschitz domain in $\mathbb{R}^n$, $n \geq 3$, with the variable viscosity coefficient. First, we introduce a parametrix and construct the corresponding parametrix-based variable-coefficient Stokes Newtonian and layer integral potential operators with densities and the viscosity coefficient in L_p-based Sobolev or Besov spaces. Then we generalize various properties of these potentials, known for the Stokes system with constant coefficients, to the case of the Stokes system with variable coefficients. Next, we show that the Robin boundary value problem for the Stokes system with variable coefficients is equivalent to a BDIE system. Then we analyse the Fredholm properties of the BDIE systems in L_p-based Sobolev and Besov spaces and finally prove their invertibility in corresponding spaces. This is a joint work with Mirela Kohr and Massimo Lanza de Cristoforis.

Investigation of nonclassical transmission problems of the thermo-electro-magneto elasticity theory for composed bodies by the integral equation method

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Investigation of nonclassical transmission problems of the thermo-electro-magneto elasticity theory for composed bodies by the integral equation method Abstract: We investigate multi-field problems for complex elastic anisotropic structures when in different adjacent components of the composed body different refined models of elasticity theory are considered. In particular, we analyse the case when we have the generalized thermo-electro-magneto elasticity model (GTEME model) in one region of the composed body and the generalized thermo-elasticity model (GTE model) in another adjacent region. Both models are associated with Green-Lindsay's model [1], [2]. This type of mechanical problems mathematically are described by systems of partial differential equations with appropriate transmission and boundary conditions. In the GTEME model part we have six dimensional unknown physical field (three components of the displacement vector, electric potential function, magnetic potential function, and temperature distribution function), while in the GTE model part we have four dimensional unknown physical field (three components of the displacement vector and temperature distribution function). The diversity in dimensions of the interacting physical fields complicates mathematical formulation and analysis of the corresponding boundary-transmission problems. We apply the potential method and the theory of pseudodifferential equations and prove uniqueness and existence theorems of solutions to different type boundary-transmission problems in appropriate Sobolev spaces. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant number FS-18-126).
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[2] T. Buchukuri, O. Chkadua, and D.Natroshvili, Mathematical Problems of Generalized Thermo-Electro-Magneto-Elasticity Theory, Memoirs on Differential Equations and Mathematical Physics, Volume 68 (2016), 1-166.

Localized boundary-domain integral equations approach with piecewise constant cut-off function for the heat transfer equation with a variable coefficient

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Localized boundary-domain integro-differential equations (LBDIDE) systems associated with the Dirichlet and Robin boundary value problems (BVP) for the stationary heat transfer partial differential equation (PDE) with a variable coefficient are obtained and analyzed. The parametrix is localized by a characteristic function of a ball of radius ε which is not a smooth cut-off function in the whole space. The main results of the present paper are equivalence theorems of the LBDIDE systems to the original variable-coefficient BVPs and unique solvability of the LBDIDE systems in the corresponding Sobolev spaces.
This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSF) (Grant number FS-18-126).

Rescaling and Trace Operators in Fractional Sobolev Spaces on Bounded Lipschitz Domains with Periodic Structure

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This paper presents rescaling of the trace operator acting on functions from fractional Sobolev type spaces and also some related results on rescaling the Bessel potential, Riesz potential, and Sobolev-Slobodetskii spaces on bounded Lipschitz domains with periodic structure.

On the convergence of Cesàro means of negative order of Vilenkin-Fourier series

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In 1971 Onnewer and Waterman establish a sufficient condition which guarantees uniform convergence of Vilenkin-Fourier series of continuous function. In my talk it is considered different classes of functions of generalized bounded oscillation and in the terms of these

classes there are established sufficient conditions for uniform convergence of Cesàro means of negative order.

On Toeplitz operators with matrix almost periodic symbols

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Fredholmness and invertibility of Toeplitz operators can be characterized fully in terms of the Wiener-Hopf factorization (or its appropriate modification) of their symbols, thus the importance of factorability criteria. Scalar continuous functions are factorable if and only if they are invertible; the result which carries over to the matrix case. Scalar almost periodic case is no different from the continuous version: factorability is equivalent to invertibility. Things change, however, in the matrix almost periodic setting: factorability still implies invertibility, but not the other way around. Moreover, the factorability criterion in this setting is still unknown. In our talk, we will describe several cases in which efficient necessary and sufficient factorability conditions have been established. Some more specific open problems will also be discussed, time permitting.

S.21: Time-frequency Analysis and Applications

Organisers
Luis Daniel Abreu, Peter Balazs

Scope of the session: This session includes aspects of time-frequency analysis and applications in its broadest sense, including Gabor frames and multipliers, applications in physics and in the analysis of non-stationary phenomena (e. g. of acoustic signals), random point processes and spectral estimation, as well as more theoretic aspects of time-frequency analysis like classical Harmonic Analysis, wavelets and analysis in function spaces (Fock/Bergman, polyanalytic, Besov, Feichtinger's algebra, other modulation and related function spaces).

—Abstracts—

Time-frequency analysis of signals periodic in time and in frequency

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Following a well-established tradition in physics, which gained widespread attention after the 2016 Nobel Prize award to Kosterlitz-Haldane-Thouless for the discovery of *topological states of matter*, we will impose periodic boundary conditions in time and frequency, leading to a distributional finite-dimensional Hilbert space of Dirac deltas for representations in the signal domain, and to a Hilbert space of functions over a 2-Torus in the time-frequency domain. The two representations are linked by the good old Short-Time-Fourier Transform, whose action on distributional spaces has been put on firm ground by a celebrated work of Feichtinger and Gröchenig.
I will present some foundational aspects of this new theory, which is undergoing an open discussion, as traditional in the Acoustics Research Institute.

Recent Developments in Time-frequency Analysis and Applications

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We will give an overview on our research on the topic of the special session.
We start with a short introduction to time-frequency representations and the concept of frames. We will talk about the implementation of time-frequency transforms, and in particular mention how to design optimal windows for reconstruction. We will present ways to get adapted and adaptive time-frequency representations and how to match it to the human auditory system. We will present how to use a time-frequency representation to detect ultrasonic mouse vocalization, as well as its usage in an inpainting algorithm.
We will introduce the notion of multipliers, and talk about their mathematical properties, in particular invertibility and unconditional convergence. We will demonstrate applications to compressed sensing using random multipliers and to perceptual sparsity by using an adaptive approach.

Frame Recycling

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Grafakos and Sansing have shown how to obtain directionally sensitive time-frequency decompositions in $L^2(\mathbb{R}^n)$ based on Gabor systems in $L^2(\mathbb{R})$; the key tool is the "ridge idea," which lifts a function of one variable to a function of several variables. We generalize their result by showing that similar results hold starting with general frames for $L^2((\mathbb{R})$, both in the setting of discrete frames and continuous frames. This allows to apply the theory for several other classes of frames, e.g., wavelet frames and shift-invariant systems. We will consider applications to the Meyer wavelet and complex B-splines. In the special case of wavelet systems we show how to discretize the representations using ϵ-nets. We will close with a short discussion of partial ridges. This is joint work with Peter Massopust (TU München), Ole Christensen (DTU Lyngby) and Florian Heinrich (University of Passau).

Current problems in Gabor Analysis

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Talking about the connection between discrete and continuous Gabor analysis.

Coorbit spaces for warped time-frequency representations

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Continuous frames on a Hilbert space $\mathcal{H}$ provide a very powerful, abstract framework for the analysis of invertible representations of functions in terms of the inner products with respect to a, possibly uncountable, dictionary. If $\mathcal{H} = \mathbf{L}^2(\mathbb{R}^d)$ or a subspace thereof, the dictionary is often chosen to be time- or frequency-invariant, i.e. to consist of all translations, respectively modulations, of a collection of generators $\{g_\omega\}_\omega \subset \mathbf{L}^2(\mathbb{R}^d)$. The actual construction of *good* continuous frames with additional structural properties may, however, be highly nontrivial. In practice, this has lead to a small number of classical construction schemes being used almost exclusively, despite often providing suboptimal performance. In the context of time-frequency analysis, these classical schemes are embodied by short-time Fourier and wavelet transforms and their, various and lesser known, extensions. The main characteristics of a time-frequency system are comprised of the *local time-frequency trade-off* and the *existence and form of discrete frames* obtained by selecting an appropriate discrete subset of the dictionary. Here, we propose the construction of time-frequency systems with almost *arbitrary local frequency resolution* by means of *warping*. The resulting *time-invariant continuous tight frames* are constructed from a prototype function $\theta\colon \mathbb{R}^d \to \mathbb{C}$, that serves the purpose of a mother "wavelet", and some weighted diffeomorphism $\Phi\colon D \to \mathbb{R}^d$, the *warping function*, that controls the local time-frequency trade-off of the resulting *warped time-frequency system* $\mathcal{G}(\theta, \Phi)$. We determine basic conditions on the warping function Φ, or more specifcally that Jacobian $D\Phi^{-1}$ of its inverse, such that a family of *coorbit spaces* can be associated with $\mathcal{G}(\theta, \Phi)$, provided that θ is smooth and decays rapidly. It is further shown that $\mathcal{G}(\theta, \Phi)$ can be sampled in an intuitive way to obtain discrete frames not only on the Hilbert space $\mathcal{H}$, but simultaneously for the entire family of considered coorbit spaces. Finally, we link the constructed function spaces with certain *decomposition spaces*. This is joint work with Felix Voigtlaender.

What is the Wigner function closest to a given square integrable function?

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One can perform "time-varying filtering" by multiplying a given Wigner function $W\psi(x,\omega)$ by a weighting function $\Gamma(x,\omega)$, such that the resulting function $W_\Gamma\psi(x,\omega) = \Gamma(x,\omega)W\psi(x,\omega)$ has some optimal time-frequency concentration. However, $W_\Gamma\psi(x,\omega)$ is not, in general, a Wigner function.
This raises the problem of determining the Wigner function closest to it. I will discuss an iterative procedure to approximate the optimal solution to this problem in a controlled way. This problem can be related to the localization problem for Wigner functions, which consists of determining the Wigner function which has the largest or smallest integral on a given measurable bounded set.

This is talk is based on joint work with J.S. Ben-Benjamin, L. Cohen, N.C. Dias and P. Loughlin.

Continuity properties of Multilinear τ-Wigner Transform

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In this work, we introduce a τ-dependent multilinear Wigner transform, W_τ, $\tau \in [0,1]$. It also includes various types of multilinear time-frequency representations, among the others the classical multilinear Wigner and Rihaczek representations and some properties for multilinear τ-Wigner transform. We then prove the boundedness properties of multilinear τ-Wigner transform both on products of Lebesgue spaces and on modulation spaces.

Planar sets of sampling for the short-time Fourier transform - From sufficient conditions to sampling bounds on polyanalytic Fock spaces

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In this talk we will discuss several aspects of the following problem: Given a window function g, under which conditions does the restriction of the short-time Fourier transform to $\Omega \subset \mathbb{R}^2$ yield a continuous frame for $L^2(\mathbb{R})$, and how does the frame bound depend on Ω. For a 'nice' class of window functions, it is known that the frame property is equivalent to Ω being relatively dense at an arbitrary scale. We will show that Ω being relatively dense at specific scales is also a sufficient condition for a greater class of window functions. Moreover, we present quantitative estimates of the lower frame bound in the case of Hermite windows using the correspondence between the short-time Fourier transform and the polyanalytic Fock space.

Compactly supported linear combinations of elements of Battle–Lemarié spline wavelet systems and decompositions in weighted function spaces

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Battle–Lemarié scaling function ϕ_n of natural order n is a polynomial spline obtained by orthogonalisation process of B_n-spline, where $B_n := B_{n-1} * B_0$ and $B_0 := \chi_{[0,1)}$. Let ψ_n be a wavelet function related to ϕ_n. A system $\{\phi_n, \psi_n\}$ is an orthonormal spline wavelet basis of order n in $L^2(\mathbb{R})$. For any $n \in \mathbb{N}$ elements ϕ_n and ψ_n have exponential decay and unbounded supports. The main result of this work is an algorithm resulting in functions Φ_n and Ψ_n which are particular finite linear combinations of integer shifts of ϕ_n and ψ_n, respectively. The crucial point is that Φ_n and Ψ_n are compactly supported on $\mathbb{R}$, and for all $n \in \mathbb{N}$ systems $\{\Phi_n, \Psi_n\}$ form Riesz bases in $L^2(\mathbb{R})$. Moreover, Φ_n and Ψ_n are finite linear combinations of basic splines B_n. The result is applied to wavelet decomposition of Besov and Triebel–Lizorkin function spaces with Muckenhoupt weights.
The research was supported by the Russian Science Foundation (project 19-11-00087).

Coorbit spaces associated to integrably admissible dilation groups

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The talk considers wavelet coorbit spaces associated to integrably admissible dilation groups. The quasi-regular representation of such groups is integrable, but not necessarily a discrete series representation. Results that will be presented are the existence of smooth, admissible vectors and the realization of the coorbit space as a decomposition space relative to an induced cover. The talk is based on joint work with Hartmut Fuehr (RWTH Aachen).

Invertibility of frame operators on Besov-type decomposition spaces

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We derive an extension of the Daubechies-Walnut criterion for the invertibility of frame operators. The criterion concerns general reproducing systems (Gabor systems, wavelets, shearlets, etc.) and Besov-type decomposition spaces as introduced by Feichtinger and Gröbner. As an application, we conclude that L^2 frame expansions associated with reproducing systems on sufficiently fine lattices extend to Besov-type spaces. This simplifies and improves on recent results on the existence of atomic decompositions, which only provide a particular dual reproducing system with suitable properties. In contrast, we conclude that the L^2 canonical dual frame expansions extend to many other function spaces, and, therefore, operations on the canonical frame expansion such as thresholding are bounded on these spaces.
This is joint work with José Luis Romero and Jordy van Velthoven.

Adaptive Fourier Decomposition in H^p Spaces

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In this paper, we study decomposition of functions in Hardy spaces $H^p(1 < p < \infty)$. First, we will give a direct application of Adaptive Fourier Decomposition(AFD) in H^2 to functions in H^p. Then, we study adaptive decomposition by the normalized Szego kernel dictionary under $p-$norm. Under the proposed decomposition procedure, we give a maximal selection principle at each step and prove the convergence.

S.22 Wavelet theory and its Related Topics

Organisers
Keiko Fujita, Akira Morimoto

Scope of the session: The theory of the mathematics is important, but it is also important to apply it to real life. We will organize the session "Wavelet theory and its related Topics" intended to discuss not only the pure mathematics, but also the applied mathematics related to the research in engineering, medicine, acoustics, and the other various fields.

—Abstracts—

Deficiency of holomorphic curves for hypersurfaces and linear systems

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In this talk, we discuss relations between the deficiency of holomorphic curves for hypersurfaces and the deficiency for the base loci of linear systems in algebraic manifolds. We also discuss methods constructing holomorphic curves with deficiencies.

Hölder regularity of anisotropic wavelets and wavelet frames: matrix approach

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The compactly supported solution (refinable function) of the functional equation

$$\varphi(x) = \sum_{k\in\mathbb{Z}^s} c_k \varphi(Mx-k), \quad x \in \mathbb{R}^s, \quad c_k \in \mathbb{R}, \quad M \in \mathbb{Z}^{s\times s},$$

can generate systems of multivariate wavelets or frames. In the univariate case, $M \geq 2$ is an integer, there are several efficient methods for determining the regularity of refinable functions. One of them, the so-called *matrix approach*, yields the Hölder exponent of $\varphi \in C(\mathbb{R})$ and, in addition, provides a detailed analysis of the modulus of continuity of φ and of its local regularity. The generalization of the matrix approach to the multivariate case turned out to be a difficult task in the case of general dilation matrices M. The special case of isotropic dilation M (all eigenvalues of M are equal in the absolute value) is currently fully understood. In this talk we discuss the challenges of the anisotropic case, i.e. the dilation matrix M has eigenvalues different in the absolute value. We show how the Hölder exponent of $\varphi \in C(\mathbb{R}^s)$ reflects the influence of the invariant subspaces of M corresponding to its different in modulus eigenvalues.

This is a joint work with V. Protasov, Moscow State University, Russia.

The Template-based Procedure of Biorthogonal Ternary Wavelets for Curve Multiresolution Processing

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Curve multiresolution processing techniques have been widely discussed in the study of subdivision schemes and many applications, such as surface progressive transmission, compression, etc. The ternary subdivision schemes are the more appealing one because it can possess high symmetry, smaller topological support and certain smoothness, simultaneously. So, biorthogonal ternary wavelets are discussed in this talk, in which refinable functions are designed for cure and surface multiresolution processing of ternary subdivision schemes. Moreover, by the help of lifting techniques, the template-based procedure is established for constructing ternary refinable systems with certain symmetry, and it also gives a clear geometric templates of corresponding multiresolution algorithms by several iterative steps. Some examples with certain smoothness are constructed.

Optimization in the construction of nearly cardinal and nearly symmetric wavelets

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In this talk, we present two approaches to wavelet construction that produce nearly cardinal and nearly symmetric wavelets on the line. The first approach models wavelet construction as an optimization problem with constraints derived from the compact support, regularity, and quadrature mirror filter (QMF) conditions imposed on the integer samples of the scaling function and its associated wavelet. The objective function is set to allow for minimization of the scaling function's deviation from symmetry or cardinality. The second method is an extension of the feasibility approach by Franklin, Hogan, and Tam to allow for near symmetry or near cardinality by considering variables generated from uniform samples of the QMF, and is solved via the Douglas-Rachford algorithm. We also give examples of complex symmetric scaling functions which have been successfully obtained using this approach. Finally, we discuss the possibility of extending these constructions to multivariate wavelets..

Two-dimensional frames for multidirectional decompositions

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In this talk we consider two-dimensional frames on $L^2(\mathbb{R}^2)$. Starting with a review on an approximation theory and a frame theory, we give our recent study of wavelet frames for multidirectional expansion of multivariate functions that have anisotropic features. Results of its numerical experiments are also shown.

Some topics on the Gabor wavelet transformation

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We have studied the blind source separation problem by using some wavelet transformations. Since the Gabor function is an exponential type, we can consider the Gabor wavelet transformation not only for square integrable functions but also for analytic functionals. In this talk, we will review our previous results, and will consider some applications by means of the characteristics of the Gabor wavelet transform of analytic functionals or square integrable functions.

Subdivision schemes on a dyadic half-line

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We consider the subdivision operator on a dyadic half-line. Necessary and sufficient convergence conditions, the connection between the subdivision scheme and the refinement equation, existence criterion of a fractal curve and continuous solution of the refinement equation and some combinatorial properties of a subdivision scheme are studied. The conjecture on convergence of subdivision schemes with non-negative masks is also formulated.

The time-frequency analysis on the half space

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This talk addresses the theory of time-frequency on the space $L^2(\mathbb{R}_+)$ of square integrable functions defined on the right half real line. We will give an overview of its history, and then focus on our recent study of the frame theory based on a class of dilation-and-modulation systems in $L^2(\mathbb{R}_+)$.

Application of frame theory to shape from defocus

Anastasia Zakharova
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In this talk we study the problem of depth reconstruction from a series of defocused images. Expanding the geometrical approach describing the intensity of a defocused image as a result of acting of a linear integral operator, we interpret the defocusing operator as analysis operator of a continuous frame of a subspace of a Hilbert space. The reconstruction can then be realized with the help of an oblique frame. We show then that the initial minimization problem formulated in terms of depth and radiance can be reduced to a minimization problem involving only the depth term.

S.23 Poster Session

Organisers
not appliable

Scope of the session: The Poster Session is directed to participants which do not see themselves reflected in the above listed sessions.

—Abstracts—

Spin Structures on p-Gonal Surfaces via Divisors

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We study spin structures on Riemann and Klein surfaces in terms of divisors. In particular, we take a closer look at spin structures on hyperelliptic and p-gonal surfaces defined by divisors supported on their branch points. Moreover, we study invariant spin divisors under automorphisms and anti-holomorphic involutions of Riemann surfaces and count them. We generalize a formula that gives 2-spin divisors proved by Mumford to the case of m-spin divisors, for an even m, supported on branch points of a hyperelliptic surface.

Analytic Improvement of Frequency Characteristic of GaN-based HEMT

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Gan-based high-electron mobility transistors (HEMTs) have demonstrated microwave power performance and have a high potential to be used in electronics power high frequency. The present work is dedicated to study the physical properties of InAlN/GaN HEMT and the approximate resolution of the Poisson and Schrödinger equations to determine the density of the two dimensional electron gas. The influences of the InAlN barrier thickness and the layer compositions on the properties of the InAlN / GaN heterojunction have been demonstrated. The improvement of drain current contribute to the improvement of the sheet concentration of two dimensional electron gas produced at the interface of InAlN barrier and GaN channel layers by spontaneous and piezoelectric polarizations. In other hand, the increasing of drain current can improve its transconductance which contribute to the enhancement of the device's frequency performance. The 2DEG density and strain between InAlN and GaN can be controlled by modulating the In and Al molar fraction. A thinner InAlN barrier layer with an accurate In and Al molar fraction can increase the 2DEG density at the interface of InAlN/GaN HEMT. By using a thinner barrier layer, InAlN/GaN HEMT gives a much higher drain current of 1.63A/mm due to its higher two electron gas density of 6.7e13 cm-2.

Join work with Adil Saadi and Seddik Bri.

Harmonic Analysis and the topology of Spheres

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In this poster we study the harmonic analysis in homogeneous spaces, based on the theory of representations of compact Lie groups, and how we can use this theory to find topological invariants of those spaces. Specifically, it deals with the case of spheres $\mathbb{S}^n$, which can be seen as homogeneous spaces by means of the quotient of special orthogonal groups $\mathrm{SO}(n+1)/\ \mathrm{SO}(n)$, and explicit calculations of its Euler character are presented. The latter with the help of the Hodge decomposition theorem and the index theorem for the Dirac-de Rham operator on the sphere.

On the series with an affine sum range in a Banach space

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This talk is about the duality between the permutations and arrangements of signs for the series with terms in

infinite dimensional Banach spaces. We discuss one sufficient condition for the affinity of the sum range of a series and give some explicit constructions of the sequences, satisfying this condition in some concrete Banach spaces. We also consider some classes of bounded, linear operators, which emerge in related problems.

Colombeau generalized stochastic processes and applications

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Colombeau generalized stochastic processes are defined. Probabilistic properties of Colombeau generalized stochastic processes are investigated with emphasis on their stationarity. A necessary condition for the existence of a stationary Gaussian solution to some class of stochastic partial differential equations is given. As an example, the stationary Klein-Gordon equation driven by higher order derivatives of white noise is considered.

Exact solutions of a nonlinear equation from the theory of nonstationary processes in semiconductors

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We consider a nonlinear partial differential equation

$$\frac{\partial}{\partial t}(\Delta u - u) + \Delta u + \operatorname{div}(u \nabla u) = 0$$

where u is real-valued function depending on the one-dimensional spatial variable x and on the time variable t¿0. We construct families of exact solutions of the given equation. Solutions are found in explicit analytical form using elementary functions and special functions. The qualitative behavior of the constructed solutions is analyzed. Joint work with dr. A.I. Aristov.
The research was supported by the Program of the President of Russian Federation for Support of Young PhDs (project no. MK-1829.2018.1).

Bifurcations of invariant manifolds of nonlinear operators in Banach spaces

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We find conditions, for an invariant manifold K of an operator N_δ in a Banach space, to be asymptotically stable. There is studied the case of the loss of stability of the invariant manifold K in the direction of a one-dimensional foliation $R_{1,\delta}x, x \in U$. There are given additional conditions for the invariant manifold K to be unstable for small $\delta > 0$, with a branching off asymptotically stable invariant manifold K_δ homeomorphic to K.

Some refinements of Ostrowski's inequality and an extension to a 2-inner product space

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The purpose of this paper is to prove certain re finements of Ostrowski's inequality in an inner product space. In section 3 we study extensions of Ostrowski type inequalities in a 2-inner product space. Finally, some applications which are related to the Chebyshev functional and the Gruss inequality are presented.

Heat-like and wave-like behaviour of the lifespan estimates for wave equations with scale invariant damping and mass

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We study the upper lifespan estimates for wave equations with scale invariant damping and mass, highlighting the competition between heat-like and wave-like behaviour, the case $\delta := (\mu_1 - 1)^2 - 4\mu_2 = 0$ and a particular condition on the initial data which implies a change in the estimates.

Global solutions of 3D incompressible MHD system with mixed partial dissipation and magnetic diffusion

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In this talk, we focuses on the 3D incompressible magnetohydrodynamic (MHD) equations with mixed partial dissipation and magnetic diffusion. Our main result assesses the global stability of perturbations near the steady solution given by a background magnetic field. The stability problem on the MHD equations with partial or no dissipation has attracted considerable interests recently and there are substantial developments. The new stability result presented here is among the very few stability conclusions currently available for ideal or partially dissipated MHD equations. As a special consequence of the techniques introduced in this paper, we obtain the small data global well-posedness for the 3D incompressible Navier-Stokes equations without vertical dissipation.

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